An Effective Langrangian Approach to the Study of X, Y Particles: A few Examples

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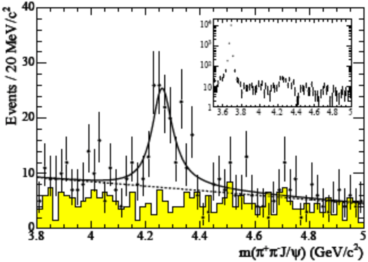
Outline

1. X(4260)
   - Effective lagrangian approach to X(4260) decay
   - Final state interactions
   - $X(4260)\psi\sigma$ and $X(4260)\psi f_0(980)$ couplings
   - Wave function renormalization and $\omega\chi_{c0}$ components
   - Discussion on X(4260)
2. A brief discussion on X(4660)
3. Future improvement and outlook

$BaBar2005 \qquad e^+ + e^- \to \gamma\,\pi^+\pi^- J/\psi$



$$m_Y = (4.259 \pm 0.008^{+2}_{-6}) GeV$$
$$\Gamma_Y = (88 \pm 23^{+6}_{-4}) MeV$$
$$J^{PC} = 1^{--}$$

$\omega\chi_{c0}$ threshold:

$$m_{\omega\chi_{c0}} = 4197\text{MeV}$$

$$\Gamma_{e^+e^-} \cdot Br(X \approx J/\psi\pi^+\pi^-) = 5.5 \pm 1.0^{+0.8}_{-0.7} eV$$

- No $e^+ + e^- \to \gamma\,\pi^+\pi^-\psi'$
- No open charm final states. (CLEO, BELLE?!)

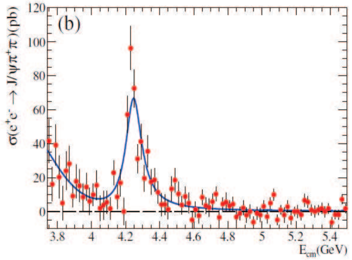


Figure: Babar2012

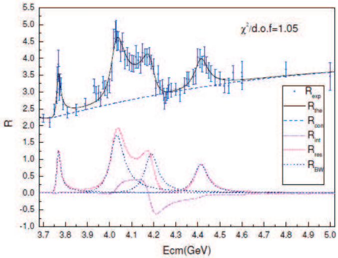
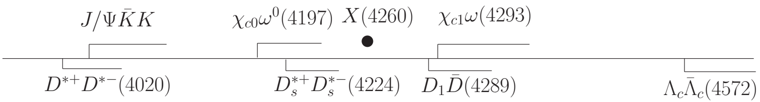


Figure: BES R value



Theoretical Models:

- $c\bar{c}$.
- Molecule: $\bar{D}_s D_{sj}(2317)$, $D\bar{D}_1(2420)$, $\omega\chi_{c1}$, $\rho\chi_{c0}$, $\Lambda_c\bar{\Lambda}_c$
- Tetraquark($c\bar{c}n\bar{n}$).
- Nonresonance, $\psi(4160)\,\psi(4415)$ interference
- Hybrid
- Non-resonant

Effective Lagrangian describing $J/\Psi\pi\pi$ decay

- $$\begin{aligned}\mathcal{L}_{\gamma X} &= g_0 X_{\mu\nu}F^{\mu\nu} \\ \mathcal{L}_{X\psi PP} &= h_1 X_{\mu\nu}\psi^{\mu\nu} < u_\alpha u^\alpha > + h_2 X_{\mu\nu}\psi^{\mu\nu} < \chi_+ > \\ &\quad + h_3 X_{\mu\alpha}\psi^{\mu\beta} < u_\beta u^\alpha > \,, \end{aligned} \tag{1}$$

- $$\begin{aligned}\mathcal{L}_1 &= \frac{4h_1}{F_\pi^2}X_{\mu\nu}F^{\mu\nu}(\partial_\rho\pi^+\partial^\rho\pi^- + \frac{1}{2}\partial_\rho\pi^0\partial^\rho\pi^0 + \partial_\rho K^+\partial^\rho K^- + \partial_\rho K^0\partial^\rho\bar{K}^0 + \frac{1}{2}\partial_\rho\eta\partial^\rho\eta) \\ \mathcal{L}_2 &= -\frac{4h_2}{F_\pi^2}X_{\mu\nu}F^{\mu\nu}(m_\pi^2\pi^+\pi^- + \frac{1}{2}m_\pi^2\pi^0\pi^0 + m_K^2K^+K^- + m_K^2K^0\bar{K}^0 + (\frac{2}{3}m_K^2 - \frac{1}{6}m_\eta^2)\eta\eta) \\ \mathcal{L}_3 &= \frac{4h_3}{F_\pi^2}X_{\mu\alpha}F^{\mu\beta}(\frac{1}{2}\partial_\beta\pi^+\partial^\alpha\pi^- + \frac{1}{2}\partial_\beta\pi^-\partial^\alpha\pi^+ + \frac{1}{2}\partial_\beta\pi^0\partial^\alpha\pi^0 + \frac{1}{2}\partial_\beta K^+\partial^\alpha K^- + \frac{1}{2}\partial_\beta K^-\partial^\alpha K^+ \\ &\quad + \frac{1}{2}\partial_\beta K^0\partial^\alpha\bar{K}^0 + \frac{1}{2}\partial_\beta\bar{K}^0\partial^\alpha K^0 + \frac{1}{2}\partial_\beta\eta^0\partial^\alpha\eta^0) \,. \end{aligned}$$

Tree level amplitude

$$i\mathcal{A}_s^{tree} = \frac{i4eg_0}{M_{J/\psi}F_\pi^2 q^2 D_X(q^2)}\bar{v}(q_1,s)\gamma_\lambda u(q_2,s')\epsilon_{\psi\theta}$$

$$\{\ [4h_1\frac{1}{2}(s-2m_\pi^2)+4h_2m_\pi^2](q^0\cdot q\ g^{\lambda\theta}-q^\theta q^{0\lambda})$$

$$+\frac{1}{2}h_3[-\frac{1}{3}\rho^2(s)q_\lambda^0 q^\theta s+\left(1-\frac{1}{3}\rho^2(s)\right)k_+^\lambda q^\theta q^0\cdot k_+ +\frac{1}{3}\rho^2(s)q^0\cdot q\ g^{\lambda\theta}s$$

$$-\left(1-\frac{1}{3}\rho^2(s)\right)g^{\lambda\theta}k_+\cdot q\ k_+\cdot q^0-\frac{1}{3}\rho^2(s)q_\lambda^0 q^\theta s+\left(1-\frac{1}{3}\rho^2(s)\right)k_+^\lambda q^{0\theta}q\cdot$$

$$+\frac{1}{3}\rho^2(s)q^0\cdot q\ g^{\lambda\theta}s-\left(1-\frac{1}{3}\rho^2(s)\right)k_+^\lambda k_+^\theta q\cdot q^0\ ]\ \},$$

$$D_X(q^2)=M_X^2-q^2-i\sqrt{q^2}(g_1k_1+g_2k_2+\Gamma(q^2)+\Gamma_0),$$

d wave tiny with very large uncertainties

Final state interactions

- $$\mathcal{A}_1 = \mathcal{A}_1^{tree}\alpha_1(s)T_{11}(s) + \mathcal{A}_2^{tree}\alpha_2(s)T_{21}(s) \ ,$$
  $$\mathcal{A}_2 = \mathcal{A}_1^{tree}\alpha_1(s)T_{12}(s) + \mathcal{A}_2^{tree}\alpha_2(s)T_{22}(s) \ ,$$
- $$\mathrm{Im}\,\mathcal{A}_1 = \mathcal{A}_1^*\rho_1 T_{11} + \mathcal{A}_2^*\rho_2 T_{21} \ ,$$
  $$\mathrm{Im}\,\mathcal{A}_2 = \mathcal{A}_1^*\rho_1 T_{12} + \mathcal{A}_2^*\rho_2 T_{22}$$
- $$\alpha_1(s) = \frac{c_0^{(1)}}{s - s_A} + c_1^{(1)} + c_2^{(1)}\, s + \cdots \ ,$$

K. L. Au, D. Morgan and M. R. Pennington, Phys. Rev. D 35 1633 (1987); D. Morgan and M. R. Pennington, Phys. Rev. D 48 1185 (1993).

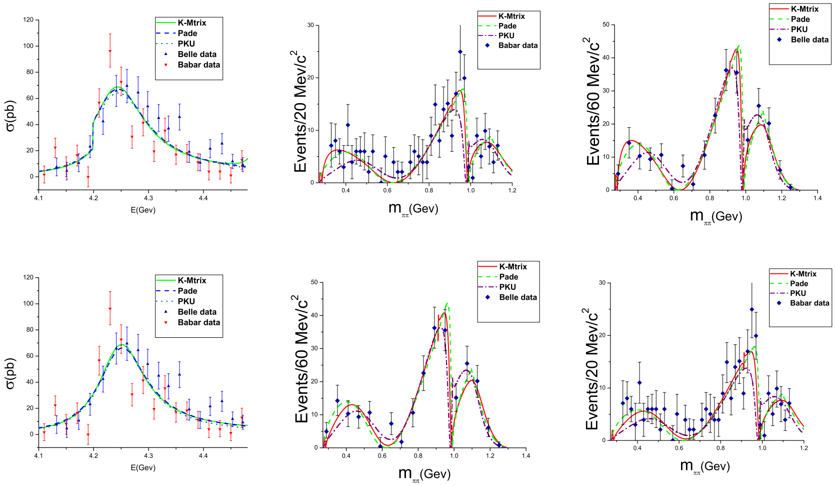
σ and $f_0(980)$ poles' location

Three test T matrices to keep theoretical uncertainties under control.

	Padé	K-matrix	PKU
sheet–II	$0.459 - 0.229i$	$0.549 - 0.230i$	$0.463 - 0.247i$
sheet–II	$0.989 - 0.013i$	$0.999 - 0.021i$	$0.980 - 0.019i$
sheet–III	-	-	-
sheet–III	-	$0.977 - 0.060i$	-

Table: Poles' location on the $\sqrt{s}$–plane, in units of GeV.

Numerical Studies



1st row: with $\omega\chi_{c0}X(4260)$ coupling g_1

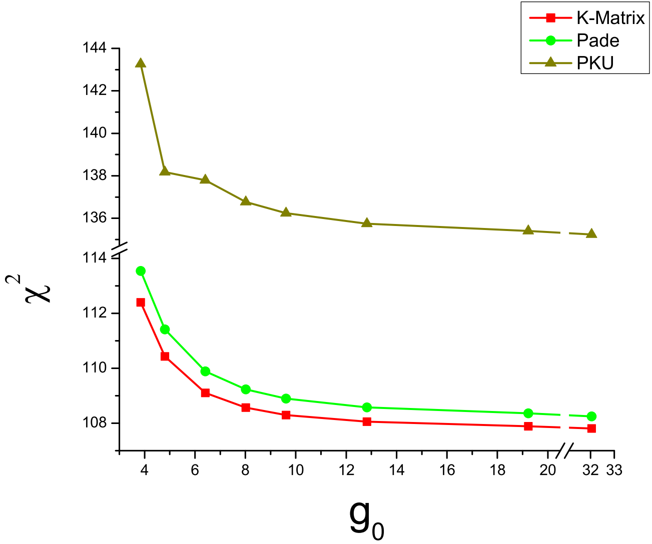
2nd row: without $\omega\chi_{c0}X(4260)$ coupling g_1

Significance Level of $g_1 \sim$ 4 σ; g_0 with a large error bar.

	fit I (Padé)	fit II (K-matrix)	fit III (PKU)
$\chi^2_{d.o.f}$	$\frac{108.3}{93-12}$	$\frac{107.8}{93-12}$	$\frac{135.2}{93-10}$
g_0(MeV)	33.416 ± 19.828	34.404 ± 19.391	34.592±14.944
g_1	0.476±0.056	0.446 ±0.048	0.507± 0.060
M_Y(GeV)	4.274 ±0.007	4.271±0.005	4.279±0.007
$N_1(pb^{-1})$	54.031± 4.958	55.326±4.647	52.494 ± 4.858
$N_2(pb^{-1})$	19.103 ±1.775	19.761±1.771	16.909±1.672
h_1	0.0542 ±0.033	0.107±0.051	0.018± 0.009
h_2	-0.554±0.496	-1.077±0.520	-0.085±0.037
h_3	-0.004± 0.048	0.0007±0.025	-0.014±0.008
$c_1^{(1)}$	0.633± 0.430	0.321 ±0.212	5.876 ± 1.908
$c_2^{(1)}$	-0.271± 0.282	-0.155 ± 0.110	-3.341 ±1.155
$c_1^{(2)}$	-0.038±0.011	-0.003 ±0.014	0
$c_2^{(2)}$	0.029±0.013	-0.0006±0.013	0

Table: Parameters given by fit to X(4260) data, assuming an X$\omega\chi_{c0}$ coupling g_1.

χ^2 Scaling



	fit I (Padé)	fit II (K-matrix)	fit III (PKU)
$\chi^2_{d.o.f}$	$\frac{108.9}{93-11}$	$\frac{108.3}{93-11}$	$\frac{136.3}{93-9}$
g_0(MeV)	9.615	9.615	9.615
g_1	0.435±0.048	0.412 ±0.043	0.450± 0.048
M_Y(GeV)	4.273 ±0.006	4.270±0.005	4.277 ±0.007
$N_1(pb^{-1})$	54.239± 4.954	55.511±5.140	52.799± 4.953
$N_2(pb^{-1})$	19.147 ± 1.807	19.795±1.935	16.951±2.053
h_1	0.187 ±0.063	0.383± 0.255	0.063± 0.065
h_2	-2.030±0.798	-3.835±2.126	-0.296±0.149
h_3	0.010± 0.151	0.002±0.0773	-0.049±0.007
$c_1^{(1)}$	0.583± 0.477	0.313 ±0.411	5.928 ± 7.26
$c_2^{(1)}$	-0.250± 0.271	-0.150± 0.195	-3.360 ± 4.354
$c_1^{(2)}$	-0.031±0.055	-0.003 ±0.022	0
$c_2^{(2)}$	0.022±0.056	-0.0006±0.021	0

Table: Parameters given by fit to X(4260) data, assuming an X$\omega\chi_{c0}$ coupling g_1 and keeping g_0 fixed.

X(4260) pole location

	Fit I	Fit II	Fit III
sheet-III	4239.1-61.2i	4239.9-57.2i	4240.9-66.1i
sheet-IV	4243.3-56.1 i	4243.2-52.9i	4246.5-59.0i

Table: The pole location of X(4260). Assuming an X$\omega\chi_{c0}$ coupling g_1, g_0 fixed at 9.62MeV.

	Fit I	Fit II	Fit III
sheet-II	4250.2-51.3i	4253.0-33.0i	4256.6-49.1i

Table: The pole location of X(4260) with g_1 switched off.

Couplings

- sheet II

$$g_{f\pi\pi}^2 = -\frac{T_{11}(z_{II})}{S'_{11}(z_{II})}, g_{X\psi f} g_{f\pi\pi} = -\frac{\mathcal{A}_1(z_{II})}{S'_{11}(z_{II})}.$$

- sheet III

$$g_{f\pi\pi}^2 = \frac{S_{22}(z_{III})}{2i\rho_1 \det S'(z_{III})} \ , \ \ g_{fKK}^2 = \frac{S_{11}(z_{III})}{2i\rho_2 \det S'(z_{III})} \ , \ \ g_{f\pi\pi}\, g_{fKK} = \frac{-T_{12}(z_{III})}{\det S'(z_{III})} \ .$$

$$g_{X\Psi f} \times g_{f\pi\pi} = -\frac{\mathcal{A}_1(z_{III}) S_{22}(z_{III})}{(\det S)'(z_{III})} + \frac{\mathcal{A}_2(z_{III}) 2i\rho_2(z_{III}) T_{21}^I(z_{III})}{(\det S)'(z_{III})}$$

$$g_{X\Psi f} \times g_{fKK} = \frac{\mathcal{A}_1(z_{III}) 2i\rho_1(z_{III}) T_{12}^I(z_{III})}{(\det S)'(z_{III})} - \frac{\mathcal{A}_2(z_{III}) S_{11}(z_{III})}{(\det S)'(z_{III})}$$

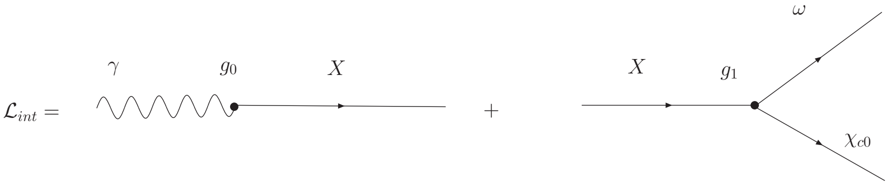
	Padé	K-matrix		PKU
	sheet-II	sheet-II	sheet-III	sheet-II
$g^2_{\sigma\pi\pi}$	-0.122 -0.127i	-0.0727 -0.169i	–	-0.147-0.133i
$g^2_{\sigma KK}$	-0.024 -0.016i	-0.063 -0.071 i	–	–
$g^2_{f_0(980)\pi\pi}$	-0.043 -0.002i	-0.071 -0.011 i	-0.100+0.022 i	-0.041 -0.005 i
$g^2_{f_0(980)KK}$	0.151-0.026 i	-0.101 +0.085 i	-0.022 -0.095 i	–

Table: Couplings of $f_0(980)$ and σ to $\pi\pi$ and $\bar{K}K$.

	Padé	K-matrix		PKU
	sheet-II	sheet-II	sheet-III	sheet-II
$g^2_{\sigma\pi\pi}$	-0.122 -0.127i	-0.0727 -0.169i	–	-0.147-0.133i
$g^2_{\sigma KK}$	-0.024 -0.016i	-0.063 -0.071 i	–	–
$g^2_{f_0(980)\pi\pi}$	-0.043 -0.002i	-0.071 -0.011 i	-0.100+0.022 i	-0.041 -0.005 i
$g^2_{f_0(980)KK}$	0.151-0.026 i	-0.101 +0.085 i	-0.022 -0.095 i	–
$g_{XJ/\psi\sigma}$	-10.102-3.311i	-6.311-4.045i	–	-9.394-3.639i
$g_{XJ/\psi f_0(980)}$	-2.845-1.287i	-4.398-0.607i	1.383+4.425i	-6.676-1.233i
$\frac{g_{XJ/\psi\sigma}}{g_{XJ/\psi f_0(980)}}$	2.510+2.299i	1.533+0.708i	–	1.458+0.276i

Table: The coupling of X(4260) to $f_0(980)$ and σ (or $f_0(600)$). The last three rows provide the results by taking $g_0 = 9.62$.

Wave function renormalization and $\omega\chi_{c0}$ components



$$g_0^R = Z_X^{1/2} g_0^B \ , \tag{3}$$

g_0^B: value of g_0 at tree level – value obtained from simple potential model calculation without considering the continuum mixing; g_0^R: the one measured by experiments.

$$\Sigma(q^2) \simeq -i\sqrt{q^2} g_1 k_1 \ , \tag{4}$$

In present approximation.

$$\Rightarrow \Sigma(\mu^2) = \frac{1}{\pi}\int_{(M_\chi+m_\omega)^2}^{\infty} \frac{\mathrm{Im}\,\Sigma(q'^2)}{q'^2-\mu^2} dq'^2 \tag{5}$$

$$\Sigma'(\mu^2) = \frac{1}{\pi}\int_{(M_\chi+m_\omega)^2}^{\infty} \frac{\mathrm{Im}\,\Sigma(q'^2)}{(q'^2-\mu^2)^2} dq'^2 \ . \tag{6}$$

Divergent in relativistic theory and hence not calculable. However, in the non-relativistic approximation, it becomes calculable for s wave interaction:

$$\mathrm{Im}\Sigma(q^2) \simeq -\sqrt{q^2} g_1 k_1 \to -\frac{g_1}{2}\sqrt{4M_\chi m_\omega}\sqrt{(q'^2-(M_\chi+m_\omega)^2)} \ . \tag{7}$$

$$Z_X^{-1} = 1 - \mathrm{Re}\,\Sigma'(\mu^2) = 1 + \frac{g_1}{2}\,\mathrm{Re}\left[\frac{\sqrt{M_\chi m_\omega}}{\sqrt{M_{th}^2-\mu^2}}\right] \ , \tag{8}$$

where $M_{th}^2 = (M_\chi + m_\omega)^2$.

If X is a bound state then

$$Z_X = \frac{1}{1 - \mathrm{Re}\,\Sigma'(\mu^2)} \simeq \frac{1}{1 + \frac{g_1}{2\sqrt{2}}\sqrt{\frac{m_R}{\epsilon}}} \ , \tag{9}$$

where we have let $\mu = M_{th} - \epsilon, \ \ m_R = \frac{M_\chi \, m_\omega}{M_\chi + m_\omega} =$ 637MeV. If X is a resonance then when $\mu \simeq M_{th} - i\Gamma/2$ one gets instead,

$$Z_X = \frac{1}{1 - \mathrm{Re}\,\Sigma'(\mu^2)} \simeq \frac{1}{1 + \frac{g_1}{2\sqrt{2}}\sqrt{\frac{m_R}{\Gamma}}} \ . \tag{10}$$

If for example taking $g_1 \simeq 0.435$ and $\Gamma \simeq 80$MeV, it is estimated $Z_{X(4260)} \simeq 0.74$. The potential model calculation of g_0 does not take into account the hadron loop effect and hence only corresponds to a 'tree level' calculation. The $\omega\chi_{c0}$ loop correction leads to that the 'tree level' value of $\Gamma_{e^+e^-}$ be reduced by a factor Z_X.

A re-study at X(3872)

One needs to look deeper into the pole structures of the scattering amplitude involving $X(3872)$.

1. For a dynamical molecule of $D^0\bar{D}^{*0}$, there is only one pole near the threshold.
2. Two nearby poles imply that it is a $c\bar{c}$ state near the threshold.

D. Morgan's pole counting mechanism!

Previous fits to the line shape of $B^+ \to XK^+$ in the $J/\Psi\pi^+\pi^-$ and $D^0\bar{D}^0\pi^0/D^0\bar{D}^{*0}$ channel give an one-pole structure (bound state or virtual bound state).

O. Zhang, C. Meng, H.Q. Zheng, Phys.Lett. B680 (2009) 453-458

$\mathcal{B} = 3 \times 10^{-4}$	$g_X(GeV)$	E_f(MeV)	$f_\rho \times 10^3$	$f_\omega \times 10^2$	Γ_c(MeV)
$\chi^2 = 4090$	4.20	-6.89	1.46	1.01	2.02 ± 1.61
$\chi^2 = 4092$	5.57	-10.3	0.74	0.53	$-$

Table: Pole positions: $E_X^{III} = M - i\Gamma/2 = -4.82 - 1.58i$MeV, $E_X^{II} = M - i\Gamma/2 = -0.20 - 0.40i$MeV (with Γ_c); $E_X^{III} = M - i\Gamma/2 = -7.66 - 0.12i$MeV, $E_X^{II} = M - i\Gamma/2 = -0.02 - 0.01i$MeV (w/o Γ_c)

Using present method:

$g_1 = 0.0937$, $m_R \simeq 966.5$MeV, $\Gamma \simeq 0.4$MeV. Bring them back to Eq. (10) one gets $Z_{X(3872)} \simeq 0.38$.

($\simeq$0.31, 0.37;Yu. S. Kalashnikova, A. V. Nefediev, Phys. Rev. D 80, 074004 (2009))

Summary

- A large coupling between X(4260) and $\omega\chi_{c0}$ is obtained, others found vanishing.
- Final state theorem applied, $|g^2_{X\Psi\sigma}/g^2_{X\Psi f_0(980)}| \sim O(10)$.
- $g_0^R = Z_X^{1/2} g_0$, $Z_X \simeq 0.8$, helpful to reduce BES bound ($\Gamma_{e^+e^-} \leq 420\text{eV}$ at 90% level). (X. H. Mo et al., Phys. Lett. B 640, 182 (2006))

A screening inter-quark potential can lower the mass of 4^3S_1 state down to 4273MeV with a $\Gamma_{e^+e^-} \simeq 970\text{eV}$, S-D mixing may reduce this number by half. (B. Q. Li, K. T. Chao, Phys. Rev. D 79, 094004 (2009))

Our analysis suggests X(4260) be a $c\bar{c}$ state renormalized by $\omega\chi_{c0}$ continuum.

Missing open charm channels?

A possible solution:

Cancelation between contribution from γ^* and X to open charms.

$\Rightarrow$ Add a constant width and a (linear) background.

	Fit I (Padé)
$\chi^2_{d.o.f}$	$\frac{108.9}{93-14}$
g_0(MeV)	9.984 $\pm$1.046
g_1	0.608$\pm$0.094
M_Y(GeV)	4.263 $\pm$0.010
Γ_0(GeV)	0.051 $\pm$ 0.008

Table: Fit results assuming X(4260) couples to $\omega\chi_{c0}$; Γ_0 and background included in the fit.

The pole position changes to 4177.3-90.0i MeV (Sheet III), 4.227.4-39.7i MeV (sheet IV). $Z_X = 0.75$; No scaling.

XDD_1 (or $\omega\chi_{c1}$) couplings?

	Fit I (Padé)
$\chi^2_{d.o.f}$	$\frac{147.0}{93-14}$
g_0(MeV)	6.836 $\pm$0.245
g_1	0.514$\pm$0.008
M_Y(GeV)	4.2118 $\pm$0.012
Γ_0(GeV)	0.017 $\pm$ 0.014
$N_1(pb^{-1})$	51.956$\pm$ 4.215
$N_2(pb^{-1})$	19.032 $\pm$0.010

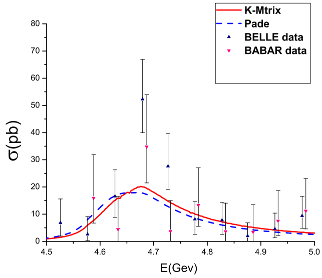
Table: Fit results assuming an equal X(4260) coupling to $\omega\chi_{c0}$ and $\bar{D}D_1$; Γ_0 and background included in the fit.

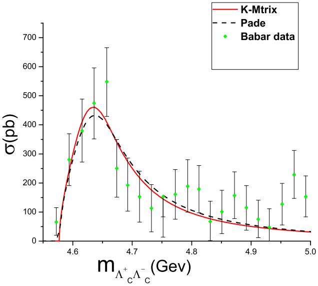
Again the fit does not support a XDD_1 coupling!

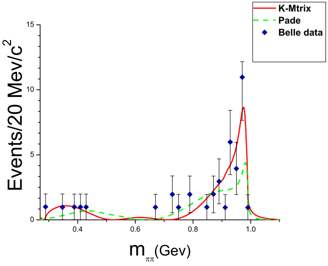
A brief discussion on X(4660)

	fit I (Padé)	fit II (K-matrix)
$\chi^2_{d.o.f}$	$\frac{47}{41-7}$	$\frac{34}{41-7}$
g_0(MeV)	7.118 $\pm$0.633	7.025 $\pm$ 0.630
g_1	2.155 $\pm$0.273	2.103 $\pm$0.275
M_Y(GeV)	4.659 $\pm$0.011	4.652 $\pm$0.010
$N_1(pb^{-1})$	17.287$\pm$ 5.861	22.257$\pm$ 6.009
h_1	-0.397 $\pm$0.129	-0.713 $\pm$ 0.172
h_2	0.537$\pm$ 0.145	0.022$\pm$0.092
$c_1^{(2)}$	1 (fixed)	1 (fixed)
$c_2^{(2)}$	-2.427 $\pm$0.709	-0.691$\pm$ 0.128

Table: Parameters given by fit to X(4660) data.


K-Mtrix
Pade
BELLE data
BABAR data
σ(pb)
E(Gev)


K-Mtrix
Pade
Babar data
σ(pb)
$m_{\Lambda^+_c \Lambda^-_c}$(Gev)


K-Mtrix
Pade
Belle data
Events/20 Mev/c^2
$m_{\pi\pi}$(Gev)

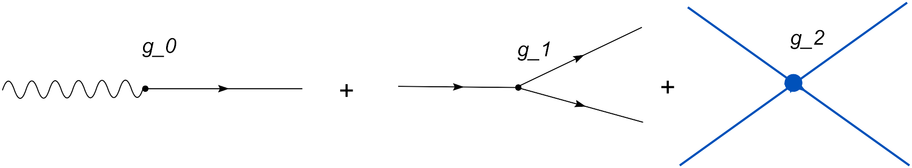
$$Z_X = \frac{1}{1 - \Sigma \mathrm{Re}\Pi_i'(\mu^2)}. \tag{11}$$

where $\Pi_1 = \Pi_{\psi' f_0(980)}$ (Three body final state interaction contribution to the spectral function integral diverges) $\Pi_2 = \Pi_{\Lambda_c^+ \Lambda_c^-}$.

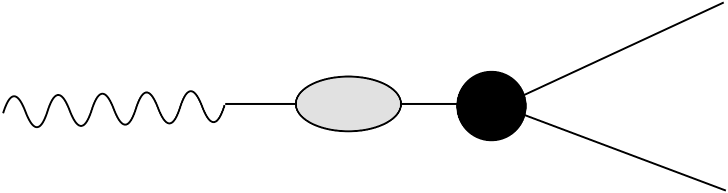
$$\Pi_i'(s_0) = \frac{1}{\pi} \int_{s_{th_i}}^{\infty} \frac{\mathrm{Im}\Pi_i(q^2)}{(q^2 - s_0)^2} dq^2, \tag{12}$$

where s_0 is square of the pole mass on the third sheet,
$\mathrm{Im}\Pi_2(q^2) = \frac{g_1^2}{12\pi}\rho_{\Lambda_c}(1 + 2\frac{M_{\Lambda_c}^2}{q^2})q^2$, $\mathrm{Im}\Pi_2(q^2) = g_{eff} k \sqrt{q^2}$. The value is $Z_X \simeq 0.7$.

Modify the effective lagrangian:



A typical production process:



$$Production\ \ Amplitude = \frac{-6g_0g_1p^2}{(p^2-m^2+i\frac{(ig_1)^2\bigcirc}{1-4ig_2\bigcirc})(1-4ig_2\bigcirc)} \qquad (13)$$

Non-perturbative renormalization

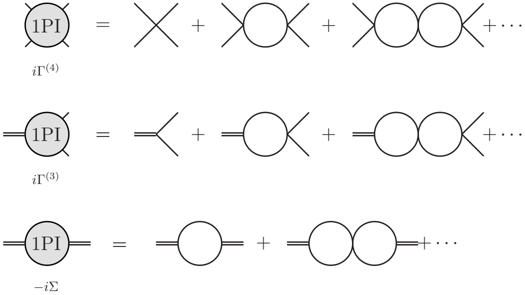


Figure: Graphic representation

Exact for NR particles!

Used for the study of $\psi(3770)$:

N. N. Achasov, G. N. Shestakov, arXiv:1208.4240

G. Y. Chen, Q. Zhao, arXiv:1209.6268

Four point function

$$i\Gamma_B^{(4)} = \frac{ig_2^0}{1 - ig_2^0 B_X(s)} = \frac{i}{\frac{1}{g_2^0} - iB_X(s)} \; , \tag{14}$$

s-wave only!

$$iB_X(s) = \frac{1}{16\pi^2}\left\{ R + 2 - \ln\frac{m_D^2}{\mu^2} + \rho(s)\ln\frac{1-\rho(s)}{1+\rho(s)} + i\pi\rho(s)\right\} . \tag{15}$$

$$\frac{1}{g_2^0} - iB_X(s) = \frac{1}{g_2} - i\tilde{B}_X(s) \; , \tag{16}$$

where

$$\tilde{B}_X(s) = B_X(s) - B_X(s_{th})$$

is finite and $s_{th} = 4m_D^2$ is the subtraction point, and

$$i\Gamma_R^{(4)} = \frac{ig_2^R}{1 - ig_2^R \tilde{B}_X(s)} \; . \tag{17}$$

Three point function

$$i\Gamma_B^{(3)} = \frac{ig_1^0}{1 - ig_2^0 B_X(s)} = \frac{i\frac{g_1^0}{g_2^0}}{\frac{1}{g_2^0} - iB_X(s)} = \frac{i\frac{g_1^0}{g_2^0}}{\frac{1}{g_2} - i\tilde{B}_X(s)}\ , \tag{18}$$

When $s = s_{th}$, let

$$\Gamma_R^{(3)}(s_{th}) = Z_X^{\frac{1}{2}}\Gamma_B^{(3)} \equiv ig_1 \tag{19}$$

then

$$\frac{g_1^0}{g_2^0} = Z_X^{-\frac{1}{2}}\frac{g_1}{g_2}\ . \tag{20}$$

Hence

$$i\Gamma_R^{(3)}(s) = \frac{ig_1^R}{1 - ig_2^R\tilde{B}_X(s)}\ . \tag{21}$$

Self energy

$iG_2 = i/(s - m_X^2 - \Sigma(s))$, and self energy reads

$$\Sigma(s) = \left(\frac{g_1^0}{g_2^0}\right)^2 \left\{ g_2^0 - \frac{1}{\frac{1}{g_2} - i\tilde{B}_X(s)} \right\}. \tag{22}$$

$$\begin{aligned} Z_X^{-1} = 1 - \Sigma(s)'\Big|_{s=m_{pole}^2} &= 1 + \left(\frac{g_1^0}{g_2^0}\right)^2 \frac{\left[i\tilde{B}_X(s)\right]'}{\left[\frac{1}{g_2} - i\tilde{B}_X(s)\right]^2}\Bigg|_{s=m_{pole}^2} \\ &= 1 + Z_X^{-1}\left(\frac{g_1}{g_2}\right)^2 \frac{\left[i\tilde{B}_X(s)\right]'}{\left[\frac{1}{g_2} - i\tilde{B}_X(s)\right]^2}\Bigg|_{s=m_{pole}^2}, \end{aligned} \tag{23}$$

hence

$$Z_X = 1 - \left(\frac{g_1}{g_2}\right)^2 \frac{\left[i\tilde{B}_X(s)\right]'}{\left[\frac{1}{g_2} - i\tilde{B}_X(s)\right]^2}\Bigg|_{s=m_{pole}^2} \tag{24}$$

be finite. Work in progress!


Thank You

Backups

- $g_0 = 6.41\text{MeV}$, $\Gamma(X \to J/\psi\pi\pi(K\bar{K})) = 21.7\text{MeV}$, $\Gamma(e^+e^-) = 93.9\text{eV}$.
- $g_0 = 9.62\text{MeV}$, $\Gamma(X \to J/\psi\pi\pi(K\bar{K})) = 9.77\text{MeV}$, $\Gamma(e^+e^-) = 211\text{eV}$.
- $g_0 = 12.82\text{MeV}$, $\Gamma(X \to J/\psi\pi\pi(K\bar{K})) = 5.55\text{MeV}$, $\Gamma(e^+e^-) = 375\text{eV}$.
- $g_0 = 19.23\text{MeV}$, $\Gamma(X \to J/\psi\pi\pi(K\bar{K})) = 2.49\text{MeV}$, $\Gamma(e^+e^-) = 845\text{eV}$.