Sensitivity study of probing relic neutrino using beta decaying target with low temperature calorimeter

2018/08/26

Outlook

1. Properties of relic neutrino

2. Total detection spectrum

3. Statistical sensitivity (analytic)

4. What to do

1. Properties of Relic neutrino

- Relic neutrino : decouple from the plasma when temperature of the universe was 1~2 MeV

- Momentum distribution : relativistic Fermi-Dirac distribution $f_\nu(p) = \left[exp\left(\frac{p}{T_\nu}\right) + 1\right]^{-1}$ with neutrino temperature today $T_{\nu,0} = 1.676 * 10^{-4} eV$

  (average kinetic energy $\approx 6.5 T_{\nu,0}\left(\frac{T_{\nu,0}}{m_\nu}\right) < \ 3.7 * 10^{-6} eV \ \ , when\ m_\nu \geq 50 meV\ (oscillation\ exp.)$)

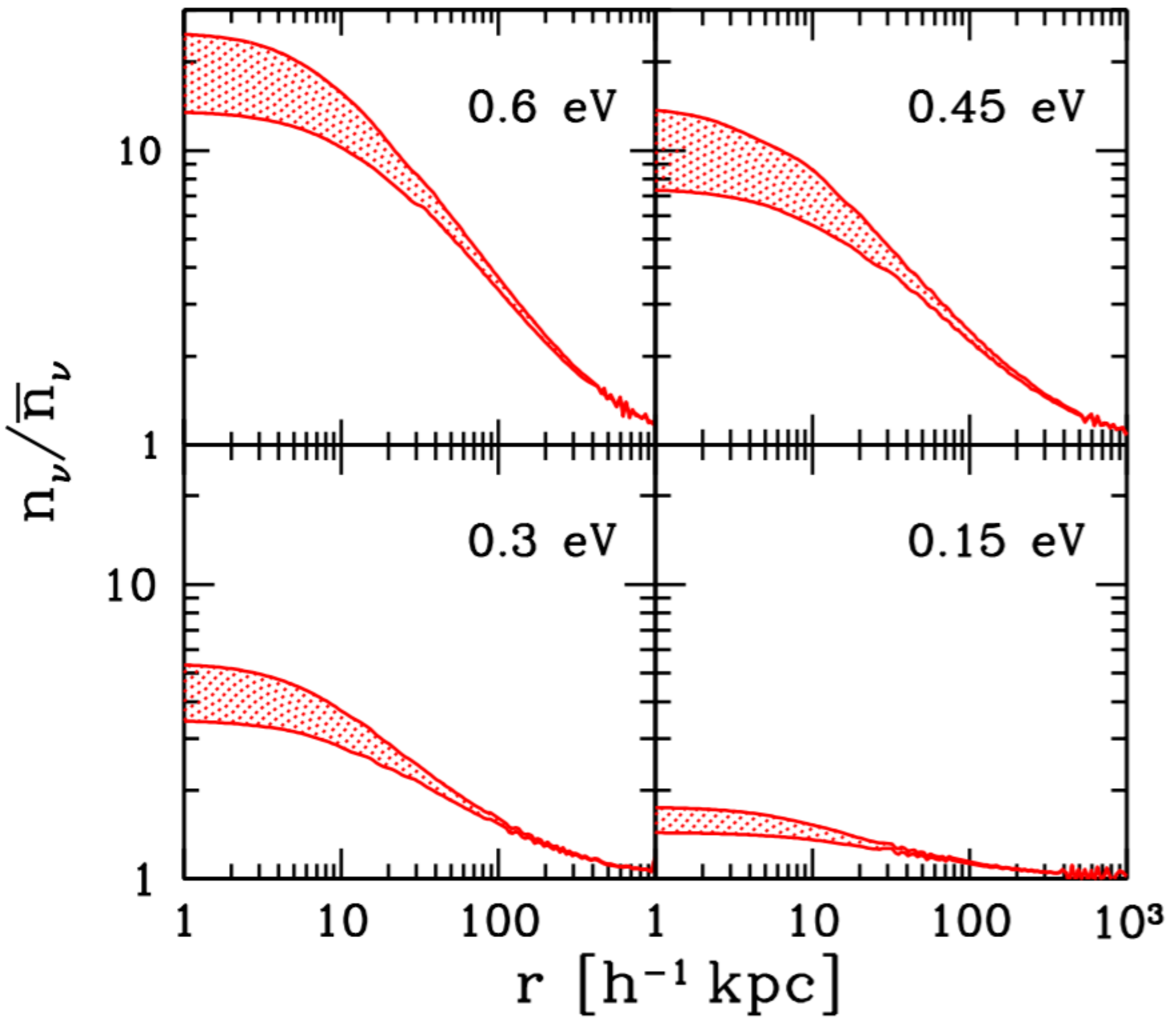
- Density per flavor, charge and helicity : $n_{\nu,0} = \frac{3\zeta(3)}{4\pi^2} {T_{\nu,0}}^3 = 56 cm^{-3} (homogenous)$ -> mass independent

  (Density enhancement factor : next page)

Because of very low kinetic energy and coupling constant of relic neutrino, it is advantageous to use beta decay nuclei as a target material -> call it NCB (neutrino capture on beta decay nuclei)

Density enhancement factor along the distance from the galactic center
(our position : 8kpc)

"Gravitational clustering of relic neutrinos and implications for their detection"



2. Total expected detection spectrum of LTD using tritium source

2.1 beta decay spectrum

- $N \rightarrow N' + e^- + \bar{\nu}_e$
- decay rate spectrum of tritium beta decay per one tritium atom : $\frac{d\lambda_\beta}{dE_e}$

$$\frac{d\lambda_\beta}{dE_e} = \frac{{G_F}^2|\mathrm{V}_{ud}|^2}{2\pi^3}|\mathcal{M}|^2 F(Z, E_e) p_e E_e p_\nu E_\nu \Theta\left(Q_\beta - E_e + m_e\right)$$

$$F(Z, E_e) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}\left(\eta = \frac{Z\alpha E_e}{p_e}\right) \; where\ Z = 2(decay\ product\ is\ He)\ and\ \alpha \sim 1/137$$

$\lvert\mathcal{M}\rvert^2 : nuclear\ \ matrix\ \ element = 5.49193$	$G_F = 1.16637 * 10^{-23} eV^2$
$\lvert\mathrm{V}_{ud}\rvert = 0.97427$	$m_e = 510999\mathrm{eV}$
$Q_\beta = 18589.8eV$	m_ν

- $\lambda_\beta = \int_{m_e}^{m_e + Q_\beta - m_\nu} \frac{d\lambda_\beta}{dE_e} dE_e = \mathbf{0.055/y}$ -> calculated half-life of tritium = $\frac{\ln(2)}{\lambda_\beta} = 12.61 year$

(experimental value = 12.32year)

2.2 NCB spectrum

- $\nu_e + N \rightarrow N^{'} + e^{-}$

- Relic neutrino capture rate spectrum : $\frac{d\lambda_{NCB}}{dE_e} = \frac{{G_F}^2|V_{ud}|^2}{2\pi}|\mathcal{M}|^2 F(Z, E_e)p_e E_e n_{\nu,0}\delta\left(E_e - m_e - Q_\beta - m_\nu\right)$

  (Dirac & non-relativistic neutrino case, for Majorana case double it)

- $\frac{\lambda_{NCB}}{n_{\nu,0}} = \frac{{G_F}^2|V_{ud}|^2}{2\pi}|\mathcal{M}|^2\{F(Z, E_e)p_e E_e\}_{E_e = m_e + Q_\beta + m_\nu} = 3.84 * 10^{-45}cm^2$ (Andrew J. Long's paper : $3.83 * 10^{-45}cm^2$).

- Capture rate per one tritium atom = $\lambda_{NCB} = \mathbf{2.03 * 10^{-25}/y}$

- 100g of tritium -> expected NCB event for 1year $\approx \mathbf{4}$

2.3 Number of events

- Assume that number of tritium atom is not affected by the NCB events or pile-up events

# of detectors	# of total tritium atom	# of tritium atom at time t	Running time	Pile-up resolving time	Energy resolution(gaussian)
N_{det}	$N_{initial}$	$\mathrm{N}(t)$	T	τ_R	$\Delta E = 2.35\sigma$

$$\mathbf{N}(\boldsymbol{t}) = \mathbf{N}(\boldsymbol{t} = \mathbf{0})\boldsymbol{e}^{-\boldsymbol{\lambda_\beta t}}$$

Number of beta events

$$= N_{det}\mathrm{N}(t = 0) * (1 - e^{-\lambda_\beta T}) = N_{initial} * \left(1 - e^{-\lambda_\beta T}\right)$$

Number of NCB events

$$= N_{det} \int_0^T \lambda_{NCB} N(t) dt = N_{det}\lambda_{NCB} \int_0^T \mathrm{N}(t = 0)e^{-\lambda_\beta t} dt = N_{initial} \frac{\lambda_{NCB}}{\lambda_\beta} (1 - e^{-\lambda_\beta T})$$

Pile-up event

- Activity of single detector at time t : $\boldsymbol{\lambda_\beta N(t)}$

- when beta decay occur at time t, probability density that next event occur at t=t+$t^{'}$ is

: $\mathrm{P}\left(\mathrm{t}, t^{'}\right) = \lambda_\beta N(t) * e^{-\lambda_\beta N(t) t^{'}}$ (normalized to 1)

- probability of next event occur within the pile-up resolving time is

$$\int_0^{\tau_R} P\left(t, t^{'}\right) dt^{'} \; = \; \int_0^{\tau_R} \lambda_\beta N(t) * e^{-\lambda_\beta N(t) t^{'}} dt^{'} \; = \; 1 - e^{-\lambda_\beta N(t)\tau_R} \; \approx \; \lambda_\beta N(t) \tau_R$$

Number of pile-up events for period T

$$= \; N_{det} * \int_0^T -\frac{dN(t)}{dt} * P_{pile-up}(\mathrm{t}) dt \; = \; N_{det} * \int_0^T \lambda_\beta N(t=0) e^{-\lambda_\beta t} * \lambda_\beta N(t=0) \tau_R e^{-\lambda_\beta t}$$

$$= \frac{1}{2} \lambda_\beta \frac{N_{initial}}{N_{det}} \tau_R N_{initial} \left(1 - e^{-2\lambda_\beta T}\right)$$

2.3 Expected detection spectrum

2.3.1 Normalized(to 1) beta, NCB, pile-up spectrum reflecting energy resolution

- $f_\beta(\sigma, E_e) = \int_{m_e}^{m_e+Q_\beta-m_\nu} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(E_e-E_e{}')^2}{2\sigma^2}} \frac{1}{\lambda_\beta} \frac{d\lambda_\beta}{dE_e}\left(E_e{}'\right) dE_e{}'$

- $f_{NCB}(\sigma, E_e) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\left(E_e-m_e-Q_\beta-m_\nu\right)^2}{2\sigma^2}}$

- $f_{pile-up}(\sigma, E_e) = \int_{2m_e}^{2(m_e+Q_\beta-m_\nu)} \left[\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(E_e-E_e{}')^2}{2\sigma^2}} \int_0^{E_e{}'} f_\beta\left(E_e{}''\right)_{\sigma=0} * f_\beta\left(E_e{}' - E_e{}''\right)_{\sigma=0} dE_e{}''\right] dE_e{}'$

2.3.2 total detection spectrum

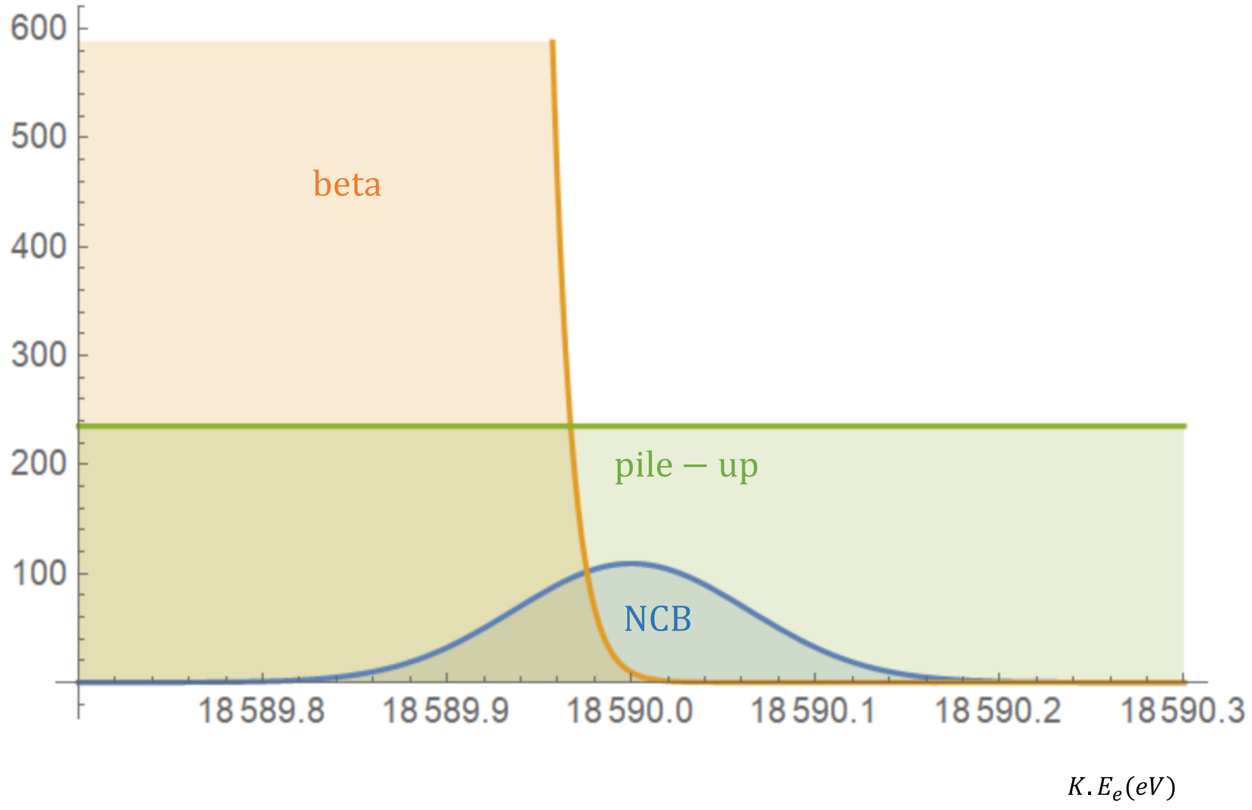
- Beta event : $N_{initial} * \left(1 - e^{-\lambda_\beta T}\right) * f_\beta(\sigma, E_e)$
- NCB event : $N_{initial} \frac{\lambda_{NCB}}{\lambda_\beta}\left(1 - e^{-\lambda_\beta T}\right)^* f_{NCB}(\sigma, E_e)$
- Pile-up event : $\frac{1}{2}\lambda_\beta \frac{N_{initial}}{N_{det}} \tau_R N_{initial}\left(1 - e^{-2\lambda_\beta T}\right)^* f_{pile-up}(\sigma, E_e)$

Total detection spectrum ($M_{tritium} = 100g, N_{det} = 1.8 * 10^{19}, \tau_R = 1 * 10^{-15}s, T = 5year$)

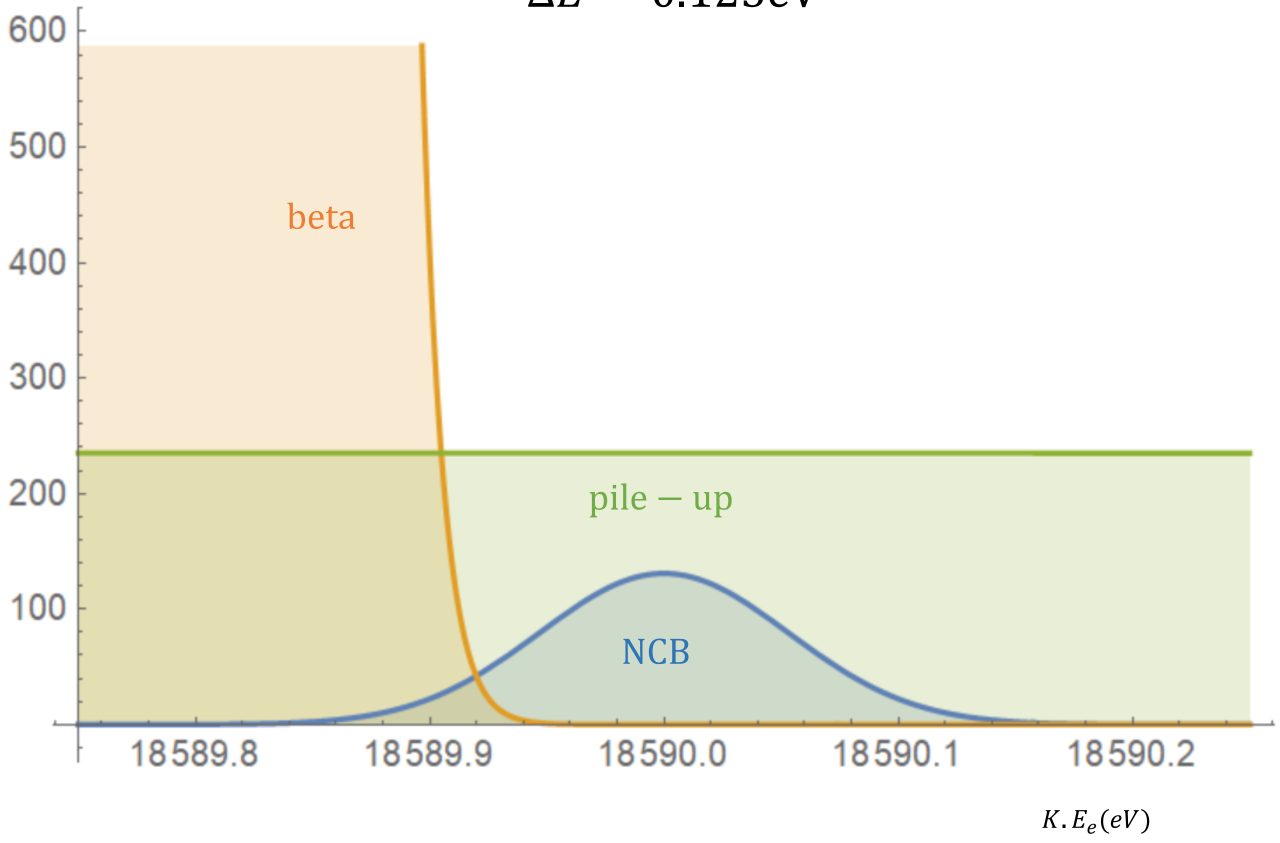
Seems unrealistic conditions

1. $m_\nu = 0.2\text{eV}$

$\Delta E = 0.15\text{eV}$

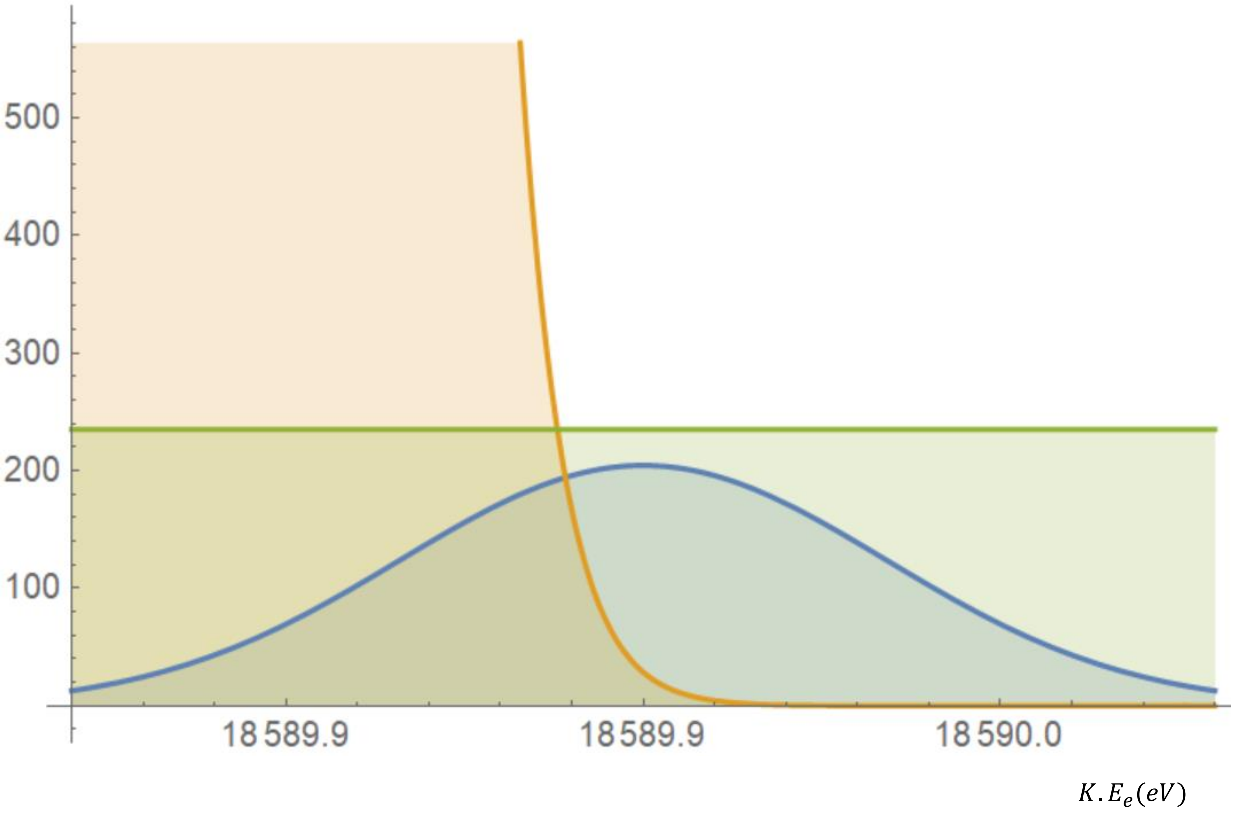


$\Delta E = 0.125\text{eV}$

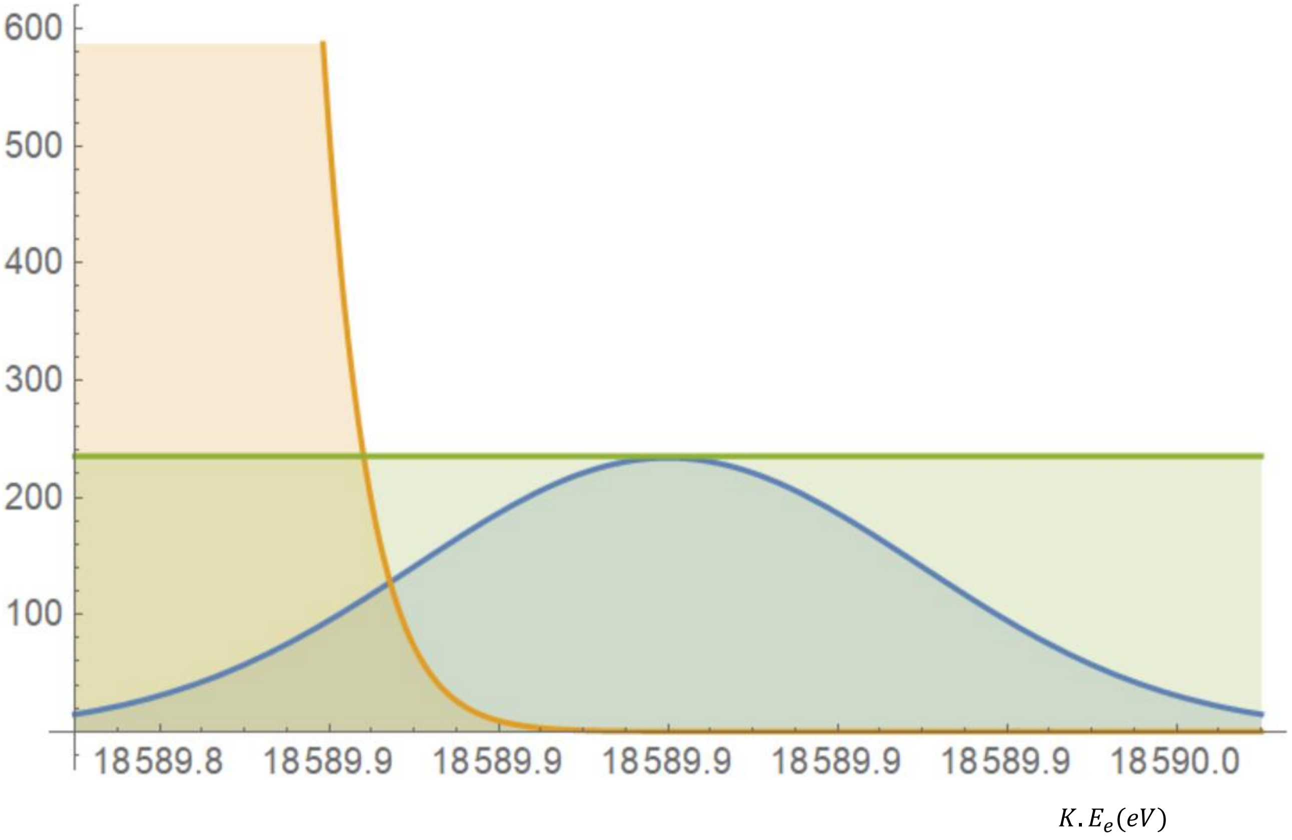


1. $m_\nu = 0.1\text{eV}$

$\Delta E = 0.08\text{eV}$



$\Delta E = 0.07\text{eV}$



3. Statistical sensitivity (analytic)

Expected number of background events

$$= b = \int_{m_e+Q_\beta+m_\nu-\frac{\Delta E}{2}}^{m_e+Q_\beta+m_\nu+\frac{\Delta E}{2}} \{N_{pile-up}(E_e, N_{initial}, N_{det}, \tau_R, T, \Delta) + N_{beta}(E_e, N_{initial}, N_{det}, \tau_R, T, \Delta)\} dE_e$$

Expected number of signal(NCB) events

$$= s = \int_{m_e+Q_\beta+m_\nu-\frac{\Delta}{2}}^{m_e+Q_\beta+m_\nu+\frac{\Delta}{2}} N_{NCB}(E_e, N_{initial}, T, \Delta) dE_e$$

Number of signal has to be bigger than the fluctuation of the number of background

95% confidence level of probing relic neutrino(3sigma result) : $\frac{s}{\sqrt{b}} = 1.96$

Reliability check

- Analytic 90% confidence limit of neutrino mass for rhenium

"Expectations for a new calorimetric neutrino mass experiment"

where N_{det} is the number of detectors and $T = N_{det} t_M$ is the exposure. This ratio must be about 1.7 for a 90% confidence limit. Therefore, in absence of background, an approximated expression for the 90% C.L. limit on m_{ν_e} – $\Sigma(m_{\nu_e})_{90}$ – can be written as [49]

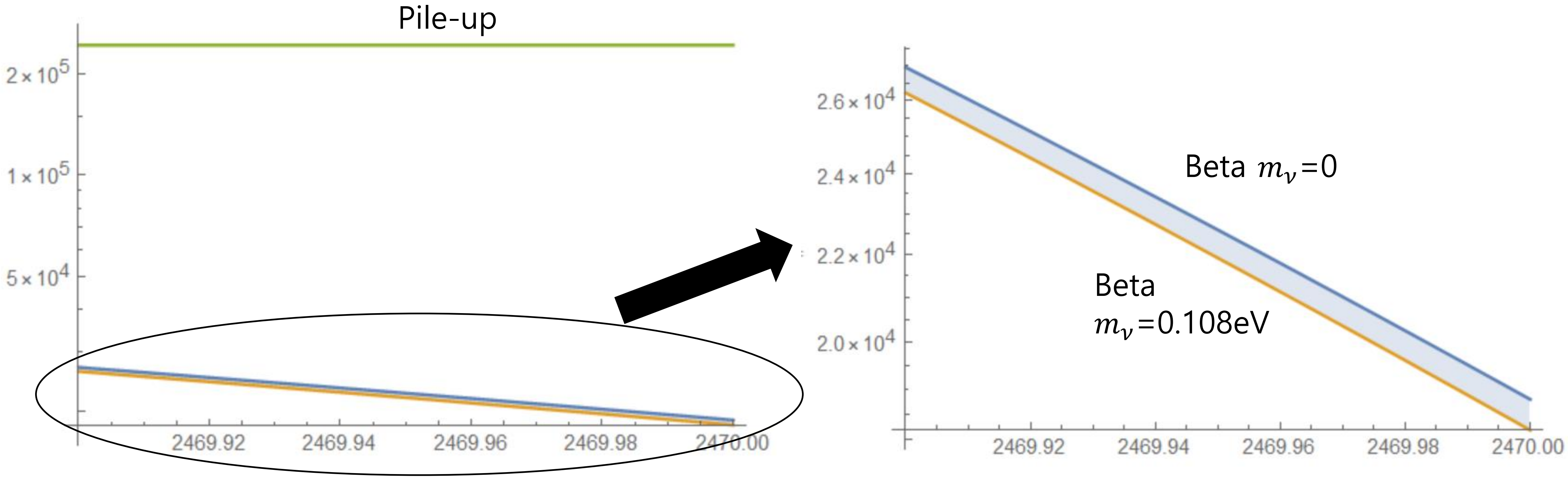
$$\Sigma_{90}(m_\nu) = 1.13 \frac{E_0}{\sqrt[4]{t_M A_\beta N_{det}}} \left[\frac{\Delta E}{E_0} + \frac{3}{10} \frac{E_0}{\Delta E} \tau_R A_\beta \right]^{\frac{1}{4}} \tag{34}$$

- 0.1eV limit of neutrino mass could be acquired when below feasible condition is met

$M_{rhenium}$	T	N_{det}	τ_R	ΔE
1.84kg	10 year	$3.3 * 10^5$	$2 * 10^{-7}$	1eV

- Use the saved formula and calculated single atom activity of rhenium $\lambda_\beta = \mathbf{1.697 * 10^{-11}/y}$

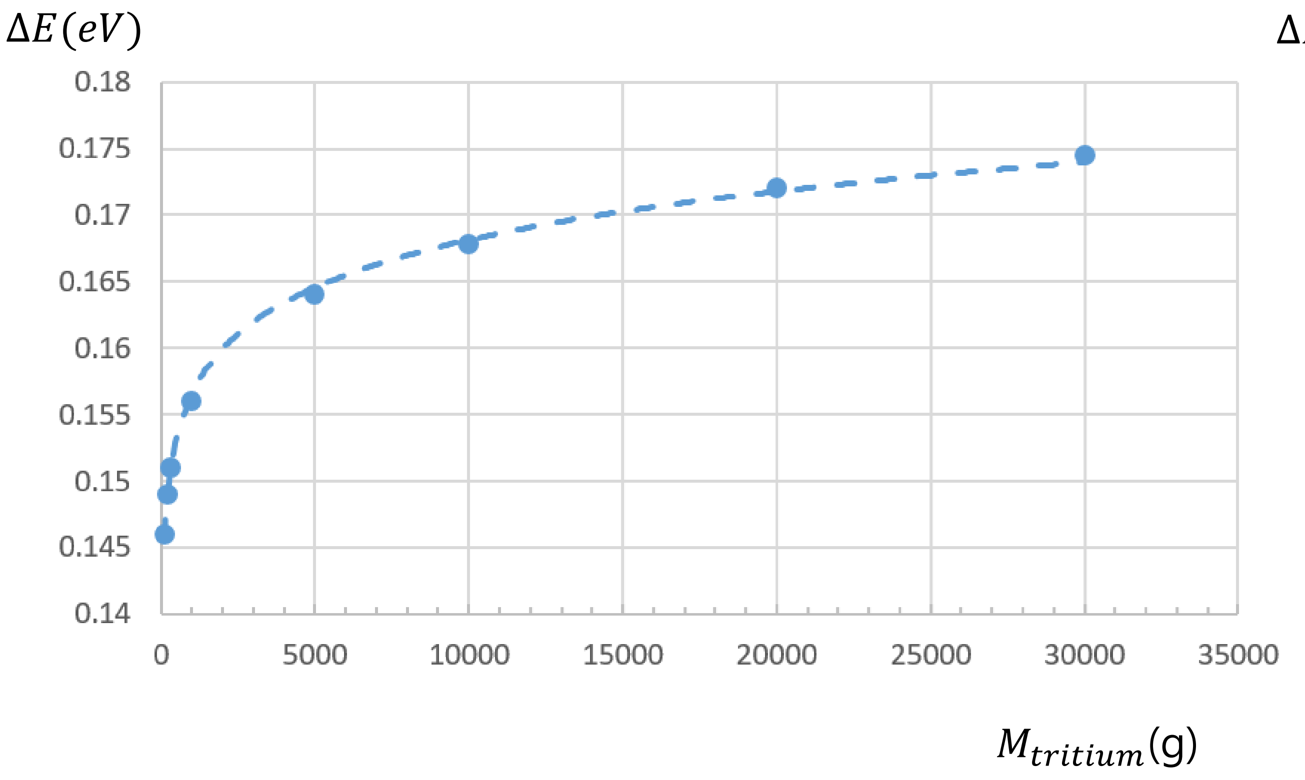
0.108eV was acquired for the 90% confidence limit

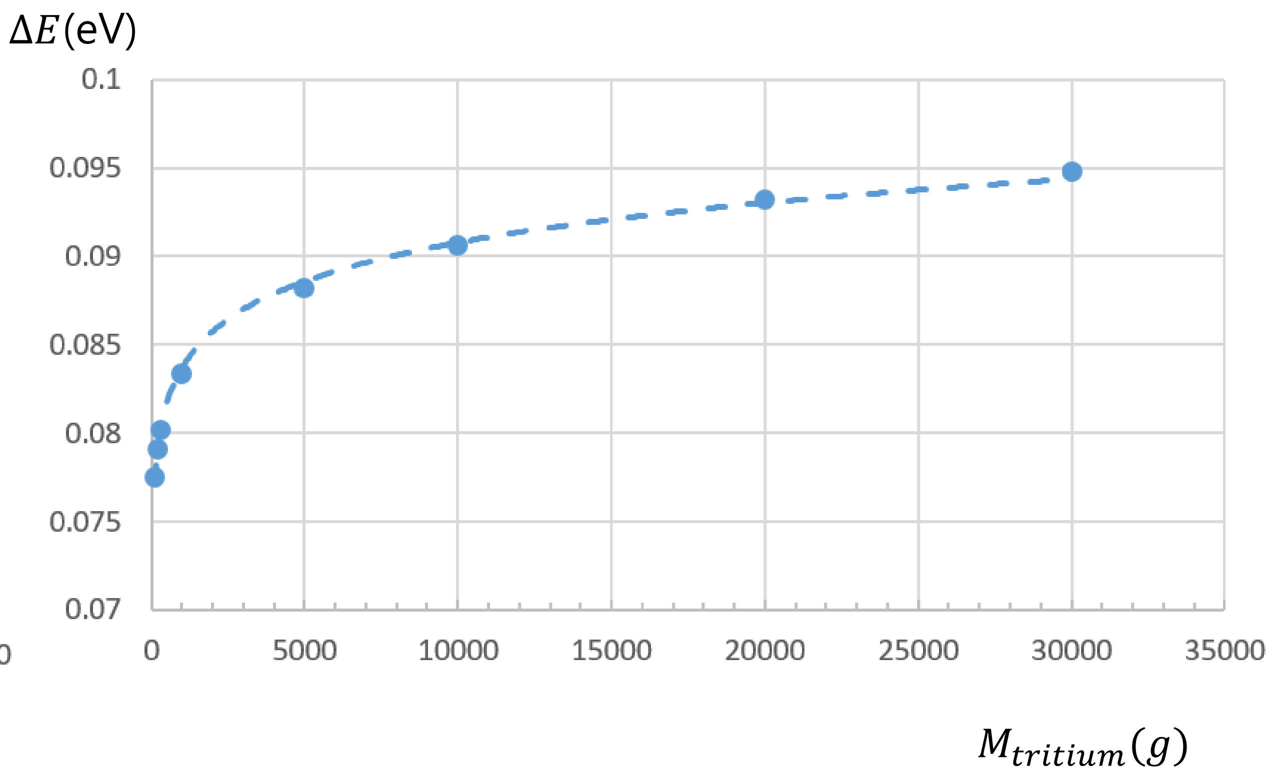

Pile-up
2×10^5
1×10^5
5×10^4
2469.92
2469.94
2469.96
2469.98
2470.00
2.6×10^4
2.4×10^4
2.2×10^4
2.0×10^4
Beta m_ν=0
Beta
m_ν=0.108eV
2469.92
2469.94
2469.96
2469.98
2470.00

Results

1. Background : beta spectrum only

Required energy resolution for total tritium mass : 100g, 200g, 300g, 1000g,…..





2. Background : pile-up spectrum only

In the case beta could be neglected(ΔE=0.1eV, m_ν=0.2eV, $M_{tritium}$=100g, T=5year) as a background

$$\frac{s}{\sqrt{b}} = \frac{N_{initial}\frac{\lambda_{NCB}}{\lambda_\beta}\left(1-e^{-\lambda_\beta T}\right)\int_{m_e+Q_\beta+m_\nu-\frac{\Delta \mathrm{E}}{2}}^{m_e+Q_\beta+m_\nu+\frac{\Delta \mathrm{E}}{2}}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{\left(E_e-m_e-Q_\beta-m_\nu\right)^2}{2\sigma^2}}dE_e}{\sqrt{\frac{1}{2}\lambda_\beta\frac{N_{initial}}{N_{det}}\tau_R N_{initial}\left(1-e^{-2\lambda_\beta T}\right)*\int_{m_e+Q_\beta+m_\nu-\frac{\Delta \mathrm{E}}{2}}^{m_e+Q_\beta+m_\nu+\frac{\Delta \mathrm{E}}{2}}f_{pile-up}(E_e)dE_e}}$$

$$= \frac{\lambda_{NCB}}{\lambda_\beta}\sqrt{\frac{N_{det}}{\lambda_\beta \tau_R}}\sqrt{\frac{1-e^{-\lambda_\beta T}}{1+e^{-\lambda_\beta T}}} * 447.693$$

$$\boldsymbol{N_{det} = 1.7 * 10^{34}\tau_R(s)}$$

When $N_{det} = 1.7 * 10^6, \tau_R = 10^{-28}s$

4. What to do

- Investigate the unavoidable energy uncertainty of LTD with Magnetic metallic sensor

 -> below 0.1eV is possible??

- Simulate the sensor signal and see how well the pile-up could be rejected

(signal shape of one beta event of $2E_e$ could be different from pile-up of E_e+E_e in absorber's temperature response)

- Vertex information?