Weekly Report(2019/04/08)

References

[1] A. Nucciotti "The use of low temperature detectors for direct measurements of the mass of the electron neutrino", Adv.High Energy Phys. 2016 (2016) 9153024 arXiv:1511.00968 [hep-ph]

[2] Hargrove, C. K.; Paterson, D. J.; Batkin, I. S "Measurement of the screening potential in 3H β decay", Physical Review C (Nuclear Physics), Volume 60, Issue 3, September 1999

[3] S. Mertens et al. "Sensitivity of Next-Generation Tritium Beta-Decay Experiments for keV-Scale Sterile Neutrinos", JCAP 1502 (2015) no.02, 020

Tritium beta spectrum

General form : $\frac{d\lambda}{dE_e} = CF(Z_f, E_e)\sqrt{{E_e}^2 - {m_e}^2}E_e\sqrt{(Q - E_e)^2 - {m_\nu}^2}(Q - E_e)\Theta(Q - m_\nu - E_e)$

- Neglects nucleus's recoil energy
- $C = \frac{{G_F}^2|V_{ud}|^2}{2\pi^3}|\mathcal{M}|^2, |\mathcal{M}|^2$: nucleus matrix element = 5.49193 $(*)$
- $F(Z_f, E_e)$: relativistic fermi function
- $m_e + Q_\beta = Q, Q_\beta = 18.598\mathrm{keV}$ $(*)$
- $(*)$: needed to check the value and error

Corrections to energy spectrum

1. Final energy state of bound electron in helium 3
2. Screening potential by electron cloud
3. Radiative correction

1. Final energy state of bound electron in helium 3 ([1],[2],[3])

- Some portion of decay energy is delivered to excitation of bound electron of helium 3
- For each state i, setting excitation probability and excitation energy to $\mathrm{w_i}$ and $\mathrm{V_i}$ respectively,
- Sudden approximation gives, $\mathrm{w_i} = \left| \int_0^\infty \mathrm{r^2 dr R_{1,0}(r)_{(\,^3H)} R_{i,0}(r)_{(\,^3He)}} \right|^2$

-> change Q to $\mathrm{Q} - V_i$ for each excitation

$$\frac{\mathrm{d}\lambda}{\mathrm{dE_e}} = \sum_i \mathrm{w_i} \mathrm{CF}(\mathrm{Z_f}, \mathrm{E_e}) \sqrt{\mathrm{E_e}^2 - \mathrm{m_e}^2} \mathrm{E_e} \sqrt{(\mathrm{Q} - \mathrm{V_i} - \mathrm{E_e})^2 - \mathrm{m_\nu}^2} (\mathrm{Q} - \mathrm{V_i} - \mathrm{E_e}) \Theta(\mathrm{Q} - \mathrm{V_i} - \mathrm{m_\nu} - \mathrm{E_e})$$

2. Screening potential by electron cloud ([2])

TABLE I. ^3H β-decay shake-up/shake-off probabilities, and screening potentials. The overlap probabilities, chemical energy shifts, and orbital excitation energy predictions are given for the β decay of atomic ^3H, for various excited levels of the daughter atom.

Final state ($^3He_n^{(*)}$)	Transition probability (α_n)	Energy shift (eV)	Excitation energy (eV)
$n=1$	0.7023	−54.46	0.0
$n=2$	0.2504	−13.52	−40.94
$n=3$	0.0131	−6.02	−48.44
$n=4$	0.0040	−3.39	−51.07
$^3He^{2+}$	0.0302	0.0	−54.46
Weighted sum	1.0000	−41.73	−12.73

- β electron feels, additionally, Coulomb potential from i^{th} sate electron cloud of helium U_i

$$U_i = \left| \int_0^\infty r^2 dr R_{i,0}(r)_{({}^3He)} \left(\frac{e^2}{r}\right) R_{i,0}(r)_{({}^3He)} \right|^2$$

- Shift electron energy E_e to $E_e - U_i$

$$\frac{d\lambda}{dE_e} = \sum_i w_i C F(Z_f, E_e - U_i)\sqrt{(E_e - U_i)^2 - {m_e}^2}(E_e - U_i)\sqrt{(Q - V_i + U_i - E_e)^2 - {m_\nu}^2}(Q - V_i + U_i - E_e)\Theta(Q - V_i + U_i - m_\nu - E_e)$$

3. Radiative correction ([1], [2], [3])

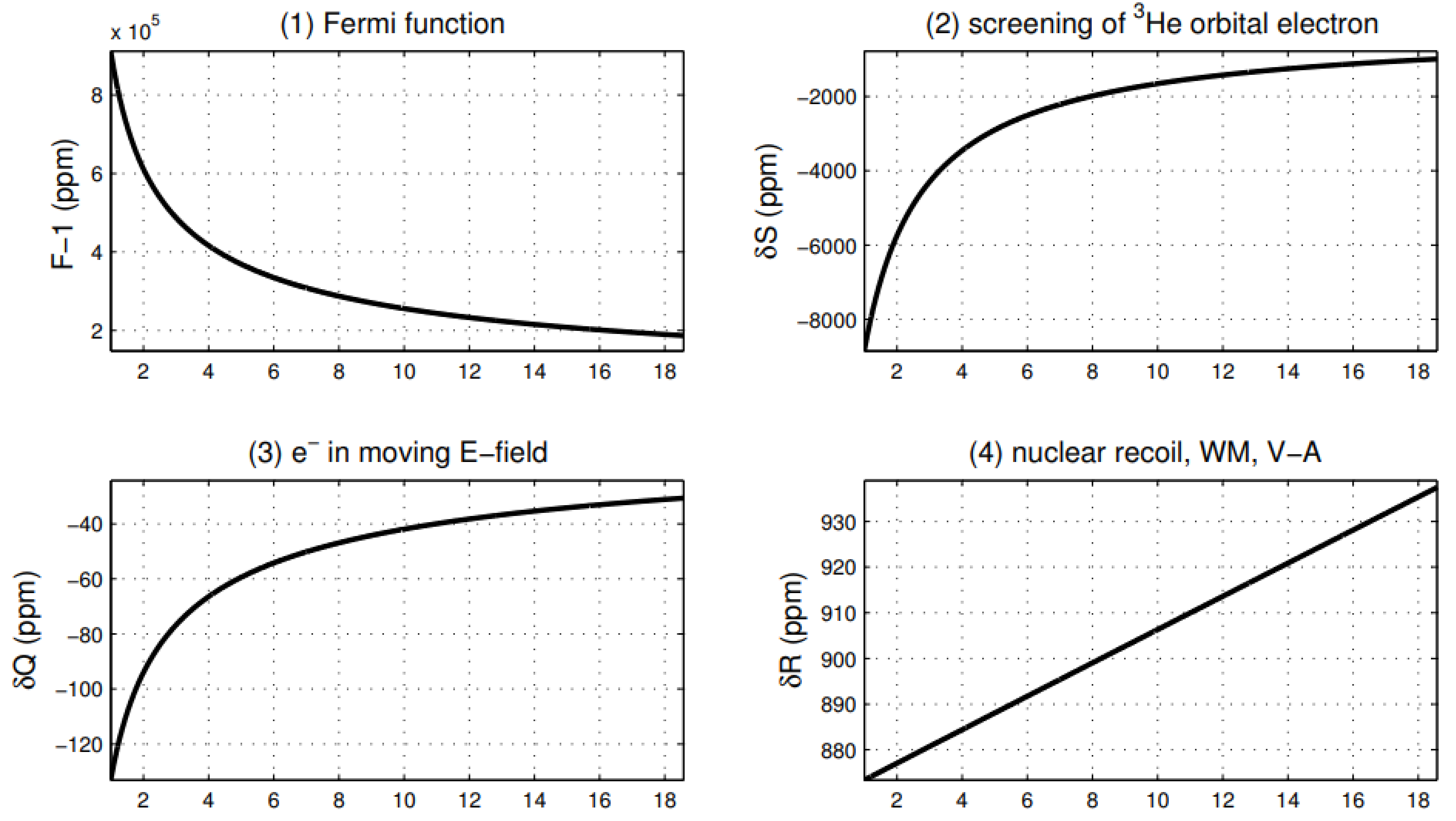
- [2] does not consider this correction for smallness of it (factor of 10^{-4}, 10^{-5} in their calorimeter) (# of event of a run : 10^8)
- In [1], it appears as multiplicatively to spectrum function $1 + \delta_R(\mathrm{E_e}, \mathrm{Z_f})$

For detector kind of calorimeter, if excited bound electron go backs to ground state within response time, detection energy will include this energy.

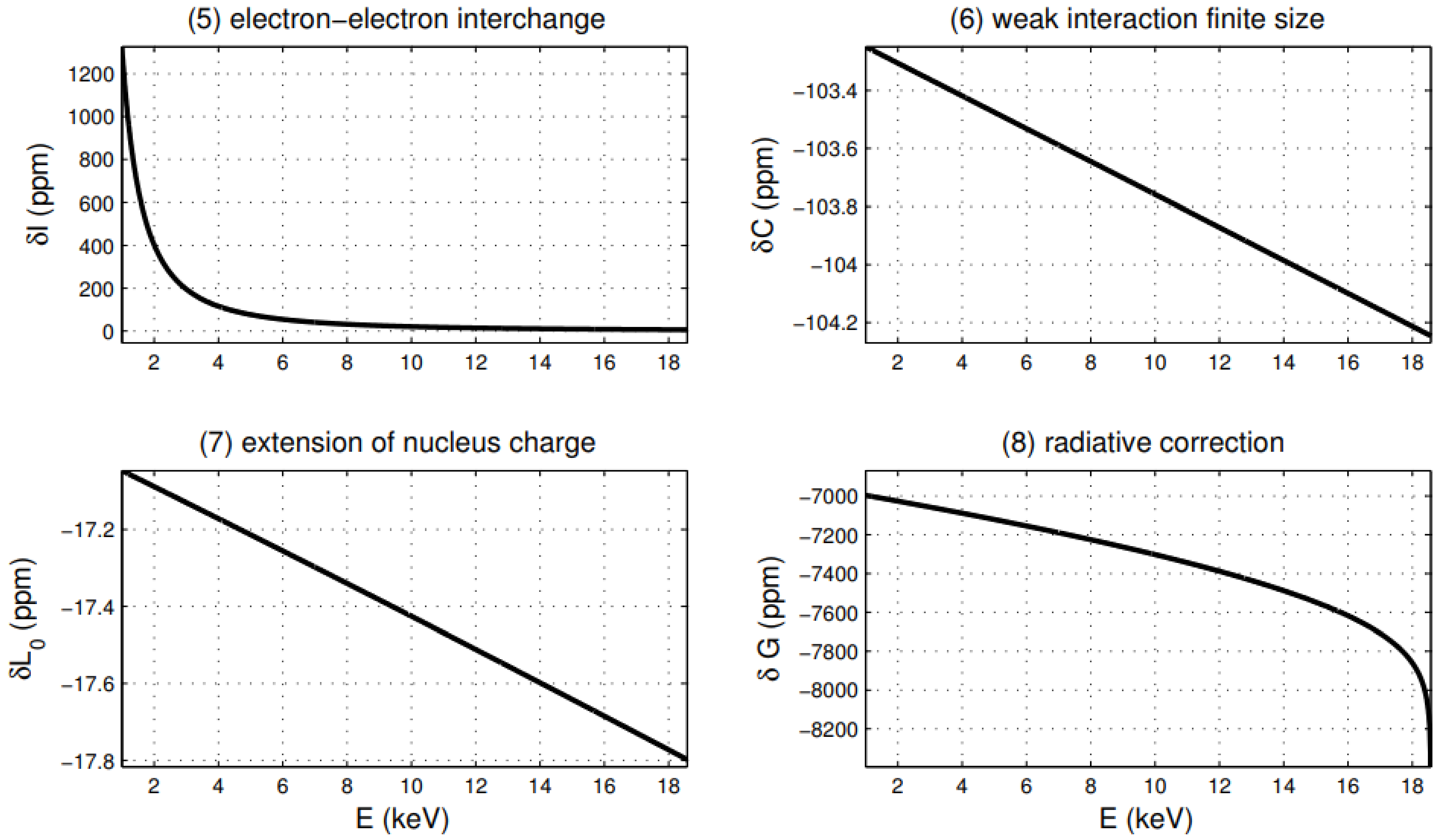
-> shift $\mathrm{E_e}$ to $\mathrm{E_e} - \mathrm{V_i}$ in final spectrum function

$$\Rightarrow \frac{d\lambda}{dE_e} = \sum_i w_i CF(Z_f, E_e - U_i - V_i)\sqrt{(E_e - V_i - U_i)^2 - {m_e}^2}(E_e - V_i - U_i)\sqrt{(Q + U_i - E_e)^2 - {m_\nu}^2}(Q + U_i - E_e)\Theta(Q + U_i - m_\nu - E_e)(1 + \delta_R(E_e, Z_f))$$

Ref [3] (T_2)



(1) Fermi function
x 10^5
F−1 (ppm)
8
6
4
2
2 4 6 8 10 12 14 16 18
(2) screening of ³He orbital electron
δS (ppm)
−2000
−4000
−6000
−8000
2 4 6 8 10 12 14 16 18
(3) e⁻ in moving E−field
δQ (ppm)
−40
−60
−80
−100
−120
2 4 6 8 10 12 14 16 18
(4) nuclear recoil, WM, V−A
δR (ppm)
930
920
910
900
890
880
2 4 6 8 10 12 14 16 18



(5) electron−electron interchange
δI (ppm)
1200
1000
800
600
400
200
0
2
4
6
8
10
12
14
16
18
(6) weak interaction finite size
δC (ppm)
−103.4
−103.6
−103.8
−104
−104.2
2
4
6
8
10
12
14
16
18
(7) extension of nucleus charge
δL0 (ppm)
−17.2
−17.4
−17.6
−17.8
2
4
6
8
10
12
14
16
18
E (keV)
(8) radiative correction
δ G (ppm)
−7000
−7200
−7400
−7600
−7800
−8000
−8200
2
4
6
8
10
12
14
16
18
E (keV)

Table 1. For each multiplicative function Ψ_i (Ψ_i = S, I, G, Q, R, C, L_0) and the final state distribution (FSD) correction we define $\delta\Psi_i = \frac{(d\Gamma/dE)^{\text{corr}}}{(d\Gamma/dE)^{\text{uncorr}}} - 1$. The variation over the whole energy spectrum is defined by $\Delta\mathcal{A} = |\delta\Psi_i(1\text{ keV}) - \delta\Psi_i(18\text{ keV})$. σ_{Ψ_i} provides a rough estimate of the uncertainty of each physical effects, obtained by varying key parameters or comparing different calculation methods as described in the 6^{th} column. Additionally, the 6^{th} column contains comments about the current status of the computation.

Correction	E=1 keV [ppm]	E=9 keV [ppm]	E=18 keV [ppm]	$\Delta\mathcal{A}$ [ppm]	Comment/Error estimation method	Ref.
δFSD(E)	1400	-635	-351175	352575	Computed only for the endpoint	[68]
σ_{FSD}	—	—	—	—		
δS(E)	-8850	-1765	-995	7860	V_0 computed only for $^3He^+$ ion	[71]
σ_S	1780	360	200		V_0 varied by $\pm 10\%$	
δI(E)	2470	45	10	1320	Excitations computed only for $^3He^+$ ion	[73]
σ_I	1145	20	5		Diff. between [72] & this work	
δG(E)	-6995	-7270	-8110	1115	Only first order considered	[77]
σ_G	25	260	830		Diff. between [77] & [77] approx.	
δQ(E)	-135	-45	-30	105	—	[75]
σ_Q	<1	<1	<1		λ_t varied by $\pm 1\%$ (3σ)	
δR(E)	875	905	935	60	—	[74]
σ_R	5	5	5		λ_t varied by $\pm 1\%$ (3σ)	
δC(E)	-105	-105	-105	1	—	[72]
σ_C	3	3	3		R varied by $\pm 5\%$	
δL_0(E)	-20	-20	-20	1	—	[72]
σ_L	6	6	6		R varied by $\pm 5\%$	