Expectations for nuclear physics at J-PARC from recent lattice QCD studies

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for HAL QCD Collaboration
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Outline

- Introduction
- Formulation --- potential (central + tensor)
- Numerical results:
  - Recent improvement for Vc and VT
  - $N\Lambda$ force (V_C + V_T)
  - $N\Sigma$ (I=3/2) force (V_C + V_T)
  - H-dibaryon at the flavor SU(3) points (Inoue)
  - ΛΛ-NΞ-ΣΣ(I=0), NΞ-ΛΣ-ΣΣ(I=1), ΣΣ(I=2) (Sasaki)
- Summary and outlook

Introduction:

- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
  - Structure of the neutron-star core,
    - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
  - H-dibaryon problem,
    - To be, or not to be,
- The project at J-PARC:
  - Explore the multistrange world,
- However, the phenomenological description of *YN* and *YY* interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological *NN* potential.

The purposes of this work

- NY forces from lattice QCD

- Spin dependence
- Potential (central + tensor)

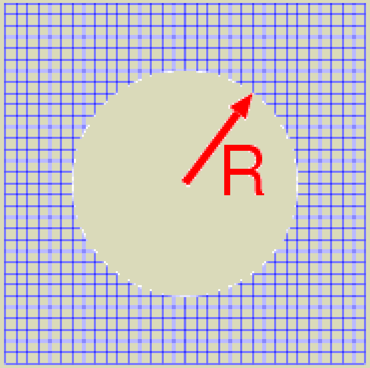
- Numerical calculation:
    - Full lattice QCD by using N_F=2+1 PACS-CS full QCD gauge configurations with the spatial lattice volume (2.86 fm)^3
    - We also use the N_F=2+1 gauge configurations by CP-PACS/JLQCD, with the spatial lattice volume (1.93 fm)^3

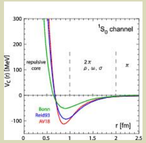
Formulation

i) basic procedure:

asymptotic region

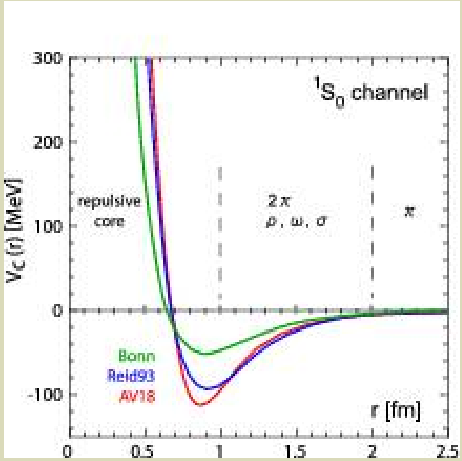
--> phase shift





ii) advanced (HAL's) procedure: interacting region

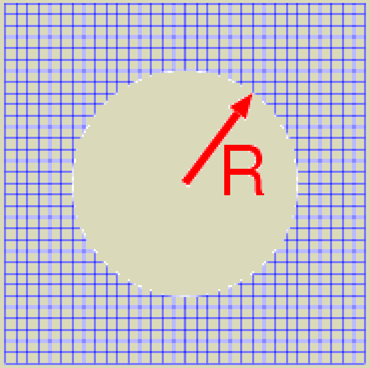
--> potential



Formulation

i) basic procedure:

An example of Luscher's formula



asymptotic region
(or temporal correlation)

--> scattering energy

--> phase shift

$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}\left(1; (kL/(2\pi))^2\right) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re\, s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991).
Aoki, et al., PRD71, 094504 (2005).

Formulation

i) basic procedure:

asymptotic region

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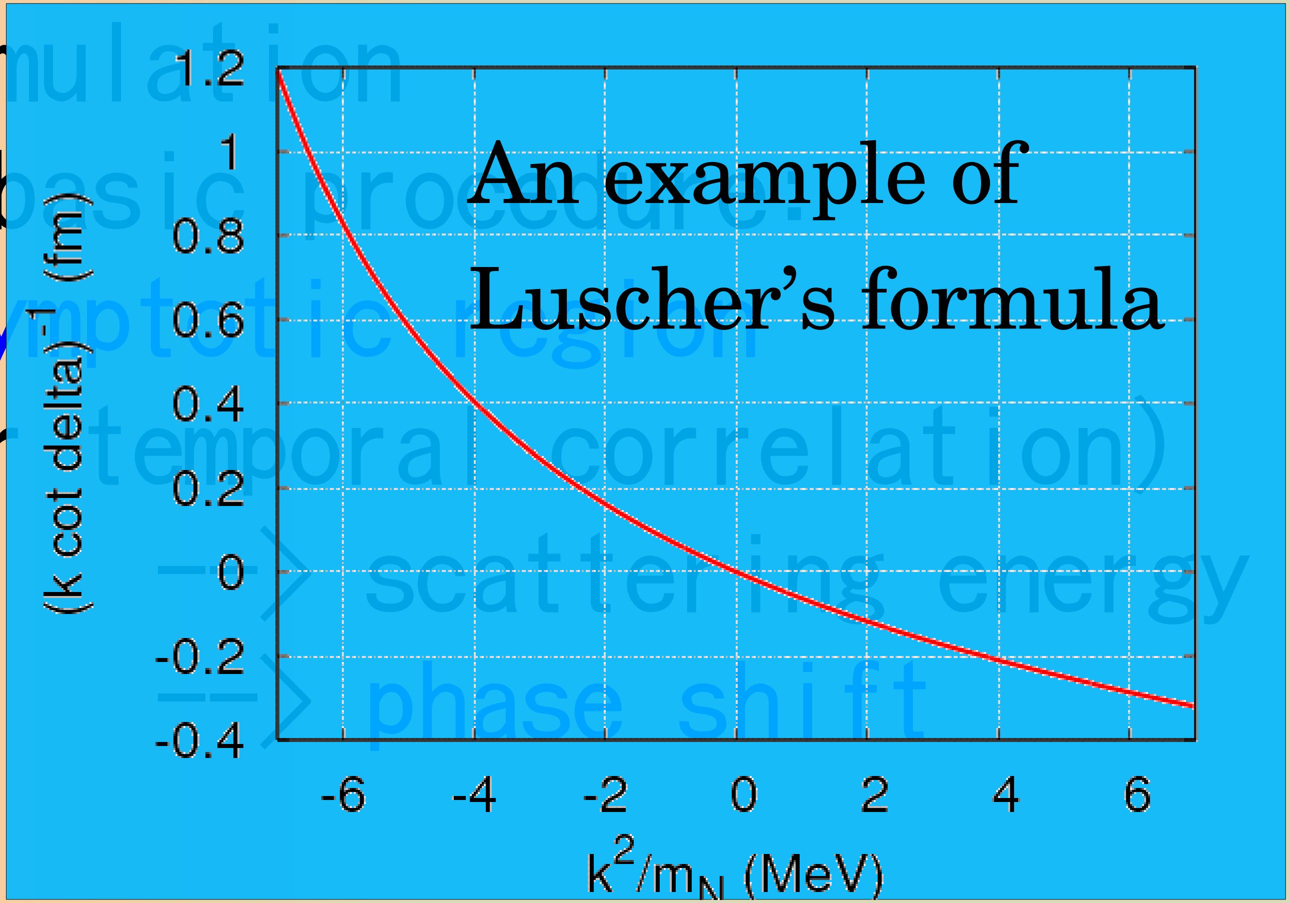

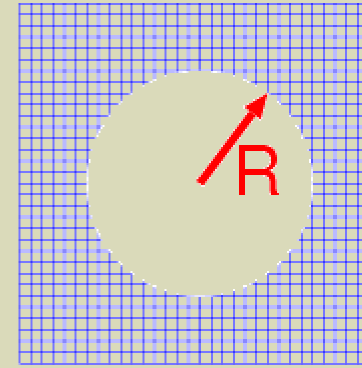

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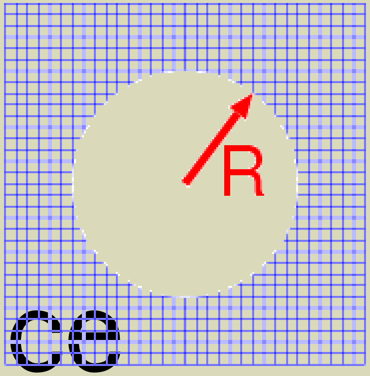
HAL formulation

ii) advanced procedure:

make better use of the lattice output ! (wave function)

interacting region

--> potential



Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., arXiv:0805.2462[hep-ph].

NOTE:

> Potential is not a direct experimental observable.

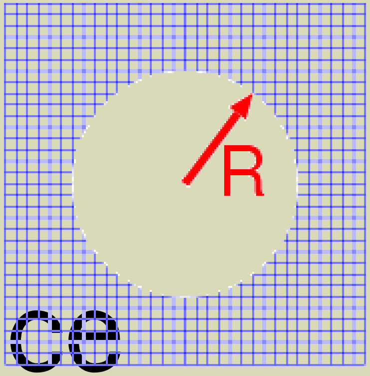
> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

HAL formulation

ii) advanced procedure:

make better use of the lattice output ! (wave function)



interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., arXiv:0805.2462[hep-ph].

=>

> Phase shift
> Nuclear many-body problems

An improved recipe for lattice potential:

- Ishii (HAL QCD, 2010); Talked LAT11 and JPS.

- Take account of the temporal correlation as well as the spatial correlation of the NBS amplitude in terms of the R-correlator:

$$R(t,\vec{r}) = \frac{C_{YN}(t,\vec{r})}{C_Y(t)C_N(t)}$$

$$R(t+\Delta t,\vec{r}) = e^{-\Delta t H} R(t,\vec{r})$$
$$= (1-\Delta t H) R(t,\vec{r})$$

- Time-dependent effective Schroedinger eq.:

$$-\frac{\partial}{\partial t} R(t,\vec{r}) = H R(t,\vec{r})$$

An improved recipe for NY potential:

- Ishii (HAL QCD, 2010); Talked LAT11 and JPS.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r}) + \int d^3r' \, U(\vec{r},\vec{r}') R(t,\vec{r}') = -\frac{\partial}{\partial t} R(t,\vec{r})$$

$$\to \frac{k^2}{2\mu} R(t,\vec{r})$$

$$U(\vec{r},\vec{r}') = V_{NY}(\vec{r},\nabla)\,\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{NY} = V_0(r) + V_\sigma(r)(\vec{\sigma}_N \cdot \vec{\sigma}_Y) + V_T(r) S_{12} + V_{LS}(r)(\vec{L}\cdot\vec{S}_+) + V_{ALS}(r)(\vec{L}\cdot\vec{S}_-) + O(\nabla^2)$$

A recipe for NΛ potential:

- The equal time BS wave function with angular momentum (J,M) on the lattice,

$$\phi^{(JM)}_{\alpha\beta}(\vec{r}) = \sum_{\vec{x}} \left\langle 0 \middle| p_\alpha(\vec{r}+\vec{x})\, \Lambda_\beta(\vec{x}) \middle| p\Lambda\,;k\,,JM \right\rangle$$

$$p_\alpha(x) = \varepsilon_{abc}(u_a(x)\, C\gamma_5\, d_b(x))\, u_{c\alpha}(x)\,,$$

$$\Lambda_\alpha(x) = \varepsilon_{abc}\left\{(d_a C\gamma_5 s_b)\, u_{c\alpha} + (s_a C\gamma_5 u_b)\, d_{c\alpha} - 2(u_a C\gamma_5 d_b)\, s_{c\alpha}\right\}$$

- The 4-point $N\Lambda$ correlator on the lattice,

$$F^{(JM)}_{\alpha\beta}(\vec{x},\vec{y},t-t_0) = \left\langle 0 \middle| p_\alpha(\vec{x},t)\, \Lambda_\beta(\vec{y},t)\, \overline{\Theta^{(JM)}_{p\Lambda}}(t_0) \middle| 0 \right\rangle$$

$$= \sum_n A^{(JM)}_n \left\langle 0 \middle| p_\alpha(\vec{x})\, \Lambda_\beta(\vec{y}) \middle| E_n \right\rangle e^{-E_n(t-t_0)}$$

$\overline{\Theta^{(JM)}_{p\Lambda}}(t_0)$ wall source at $t = t_0$

A recipe for NΛ potential:

- Ishii (HAL QCD, 2010); Talked LAT11 and JPS.

- Calculate the 4-point $N\Lambda$ correlator on the lattice,

$$\phi_{N\Lambda}(x-y)e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\Lambda_\beta(y,t)\overline{\Lambda_\beta}(0,t_0)\overline{p_\alpha}(0,t_0)\rangle$$

  - Which has the physical meanings of,
    - Create a $N\Lambda$ state and making imaginary time evolution, in order to have the lowest state of the $N\Lambda$ system.
    - Take the R-correlator $R(t-t_0, \boldsymbol{x-y})$, which can be understood as a wave function of the non-relativistic quantum mechanics.

- Obtain the effective central potential from the effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2+V(r)\right)R(t,\vec{r})=-\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\rightarrow \quad V(r)=\frac{-\frac{\partial}{\partial t}R(t,\vec{r})}{R(t,\vec{r})}+\frac{\hbar^2}{2\mu}\frac{\nabla^2 R(t,\vec{r})}{R(t,\vec{r})}$$

A recipe for NY potential: (contd.)

- For $J = 1$, ϕ comprises *S*-wave and *D*-wave,

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

where,

$$|\phi_S\rangle = \mathcal{P}|\phi\rangle = (1/24)\sum_{\mathcal{R}\in O}\mathcal{R}|\phi\rangle$$

$$|\phi_D\rangle = Q|\phi\rangle = (1 - \mathcal{P})|\phi\rangle$$

- Therefore, we have 2-component Schrödinger eq.

S-wave:

$$\mathcal{P}(T + V_C + V_T S_{12})|\phi\rangle = -\partial/\partial t\, \mathcal{P}|\phi\rangle$$

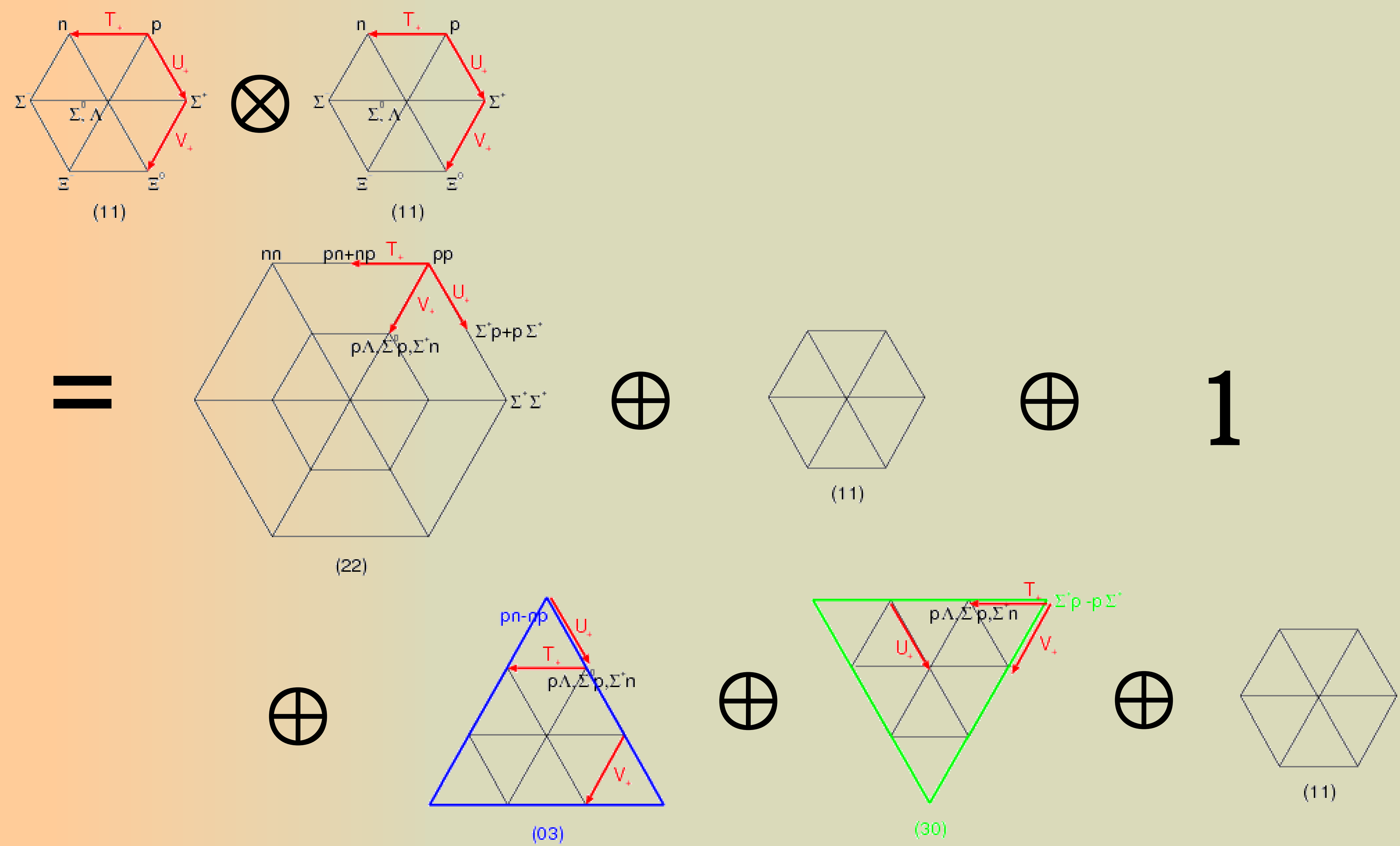
D-wave:

$$Q(T + V_C + V_T S_{12})|\phi\rangle = -\partial/\partial t\, Q|\phi\rangle$$

- Obtain the $V_C(r)$ and the $V_T(r)$ simultaneously.

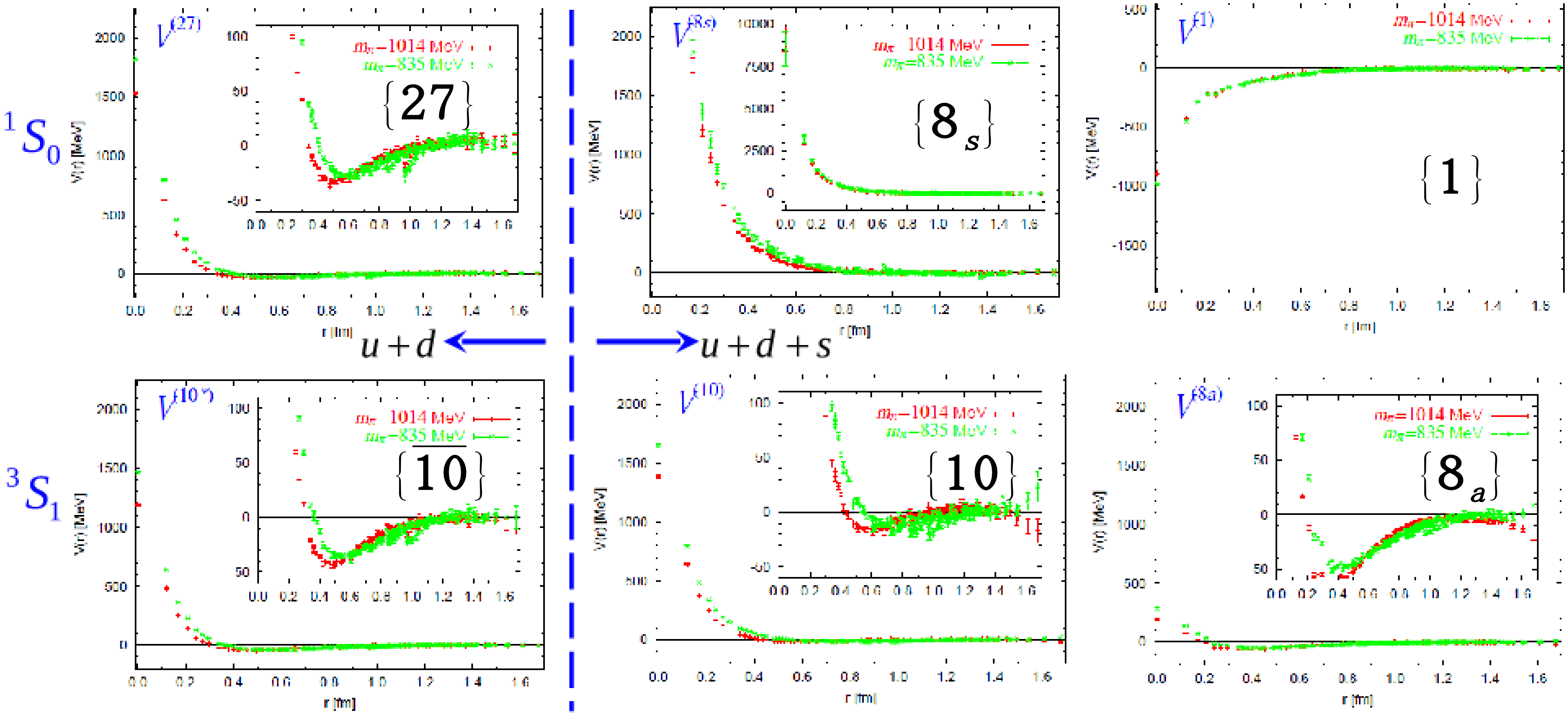
Numerical results:

$$8 \otimes 8 = 27 \oplus 8_s \oplus 1 \oplus \overline{10} \oplus 10 \oplus 8_a$$



> SU(3) limit

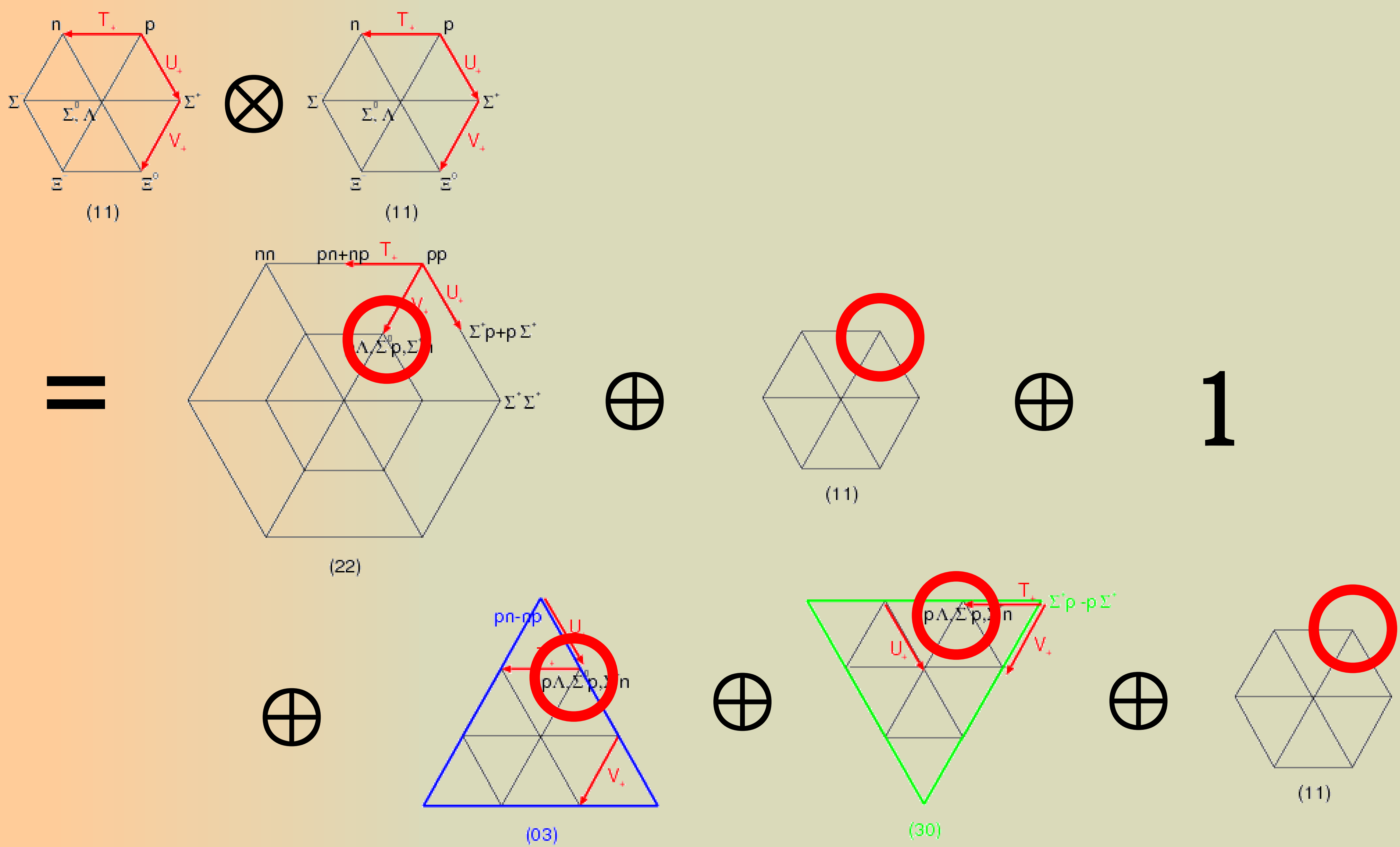
Aim: A systematic study of short range baryon-baryon interactions



- Strong flavor dependence
  - Strong repulsive core for flavor $\mathbf{8_s}$ representation.
  - All distance attraction for flavor **1** representation.
  - Weak repulsive core for flavor $\mathbf{8_a}$ representatin.
- This dependence is consistent with quark Pauli blocking picture.

Inoue, et al. (HAL QCD), PTP**124**, 591 (2010).

$$8\otimes 8 = 27\oplus 8_s \oplus 1 \oplus \overline{10} \oplus 10 \oplus 8_a$$



NΛ and NΣ scattering can access all of the flavor channels except for the flavor singlet.

Proposals for Nuclear and Particle Physics Experiments at J-PARC

P40:
Measurement of the cross sections of Σ p scatterings
Spokesperson: K. Miwa (Tohoku U., Japan)

Results

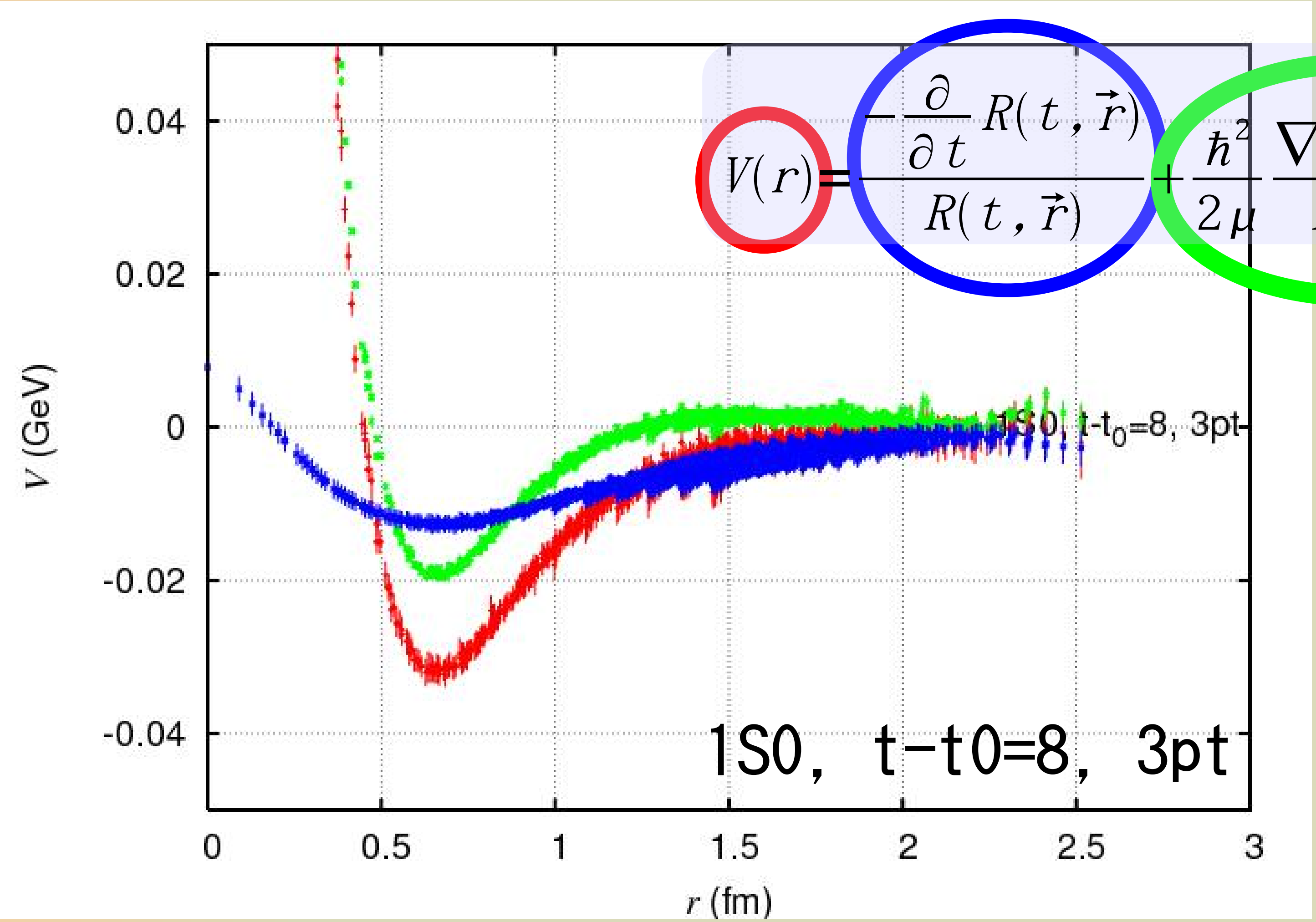
Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

- S. Aoki, et al., (PACS-CS Collaboration), PRD**79**, 034503 (2009), arXiv:0807.1661 [hep-lat].
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark action
- $1/a$ = 2.17 GeV (a = 0.0907 fm)

$(\kappa_{ud})_{N_{\rm conf}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_Ξ
2+1 flavor QCD by PACS-CS with $\kappa_s = 0.13640$ @present calc (Dirichlet BC along T)								
$(0.13700)_{609}$	700.0(4)	1108(3)	785.8(3)	1159(2)	1573(4)	1632(4)	1650(5)	1700(4)
(0.13754)	415(1)	903(5)	639.7(8)	1024(4)	1232(10)	1354(6)	1415(7)	1512(4)
Exp.	135	770	494	892	940	1116	1190	1320

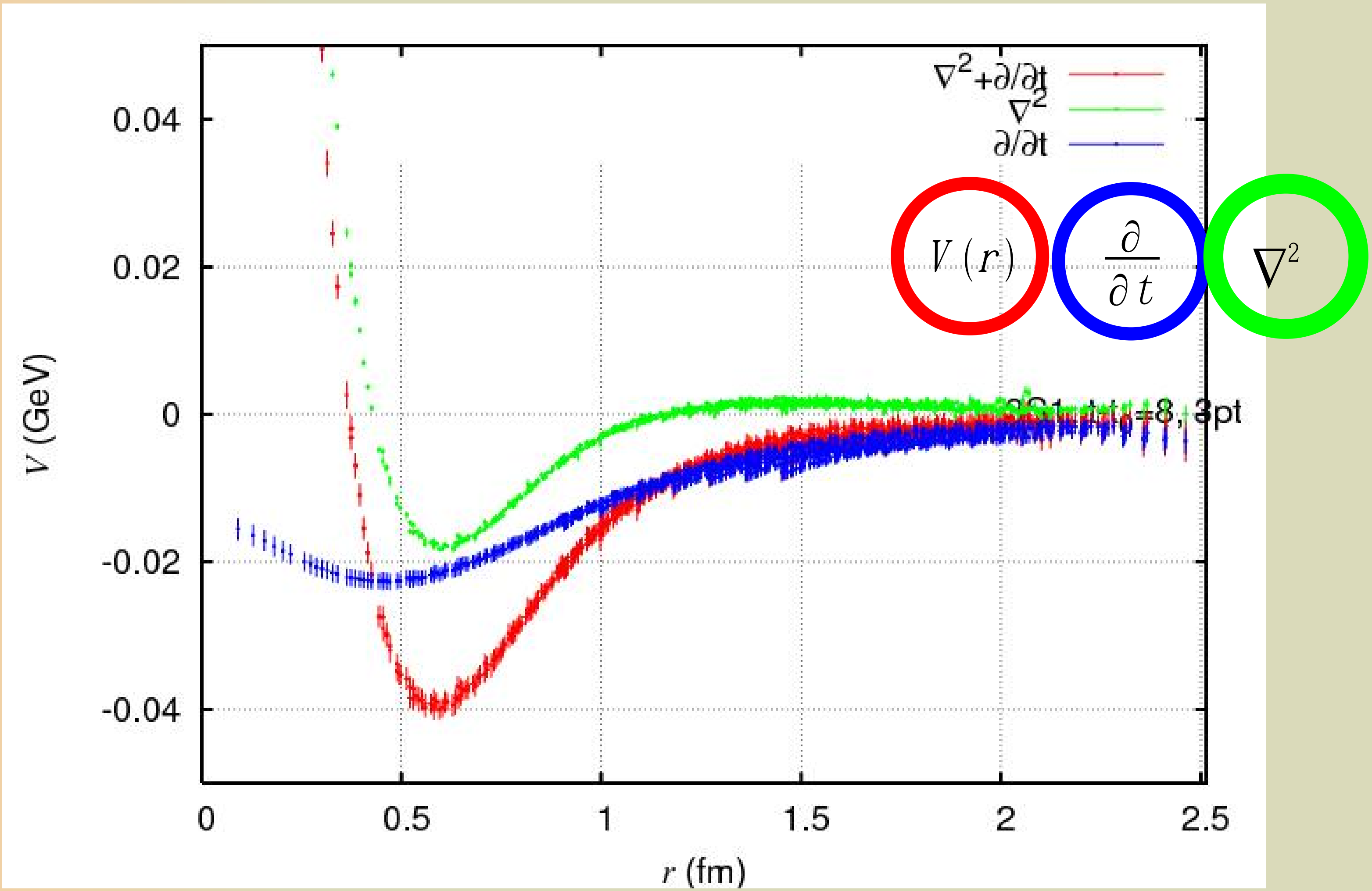
ΛN potential

$V_C(\Lambda N;\ 1S0)$

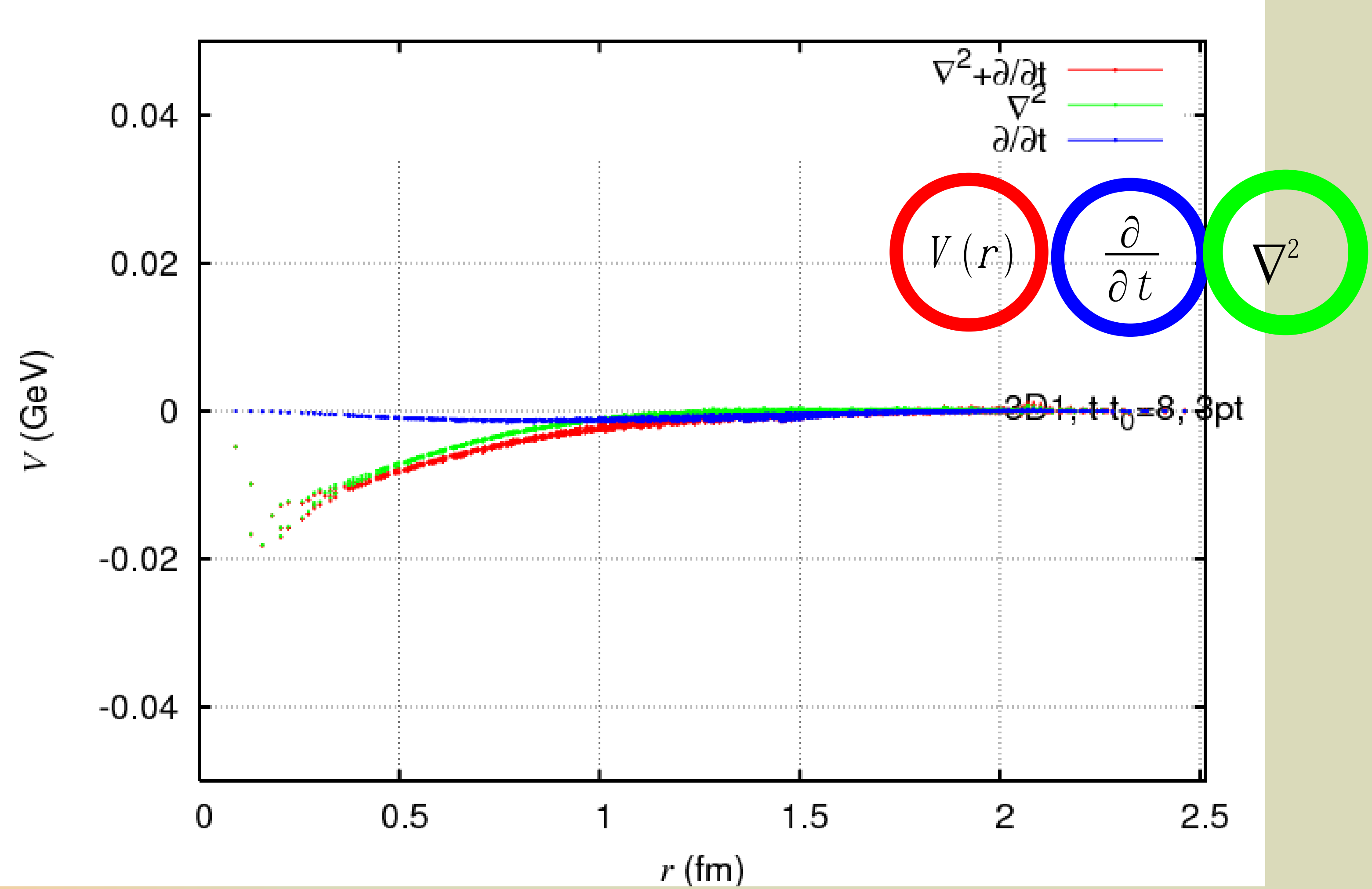


$$V(r) = \frac{-\frac{\partial}{\partial t} R(t,\vec{r})}{R(t,\vec{r})} + \frac{\hbar^2}{2\mu} \frac{\nabla^2 R(t,\vec{r})}{R(t,\vec{r})}$$

$V_C(\Lambda N;\ 3S1-3D1)$

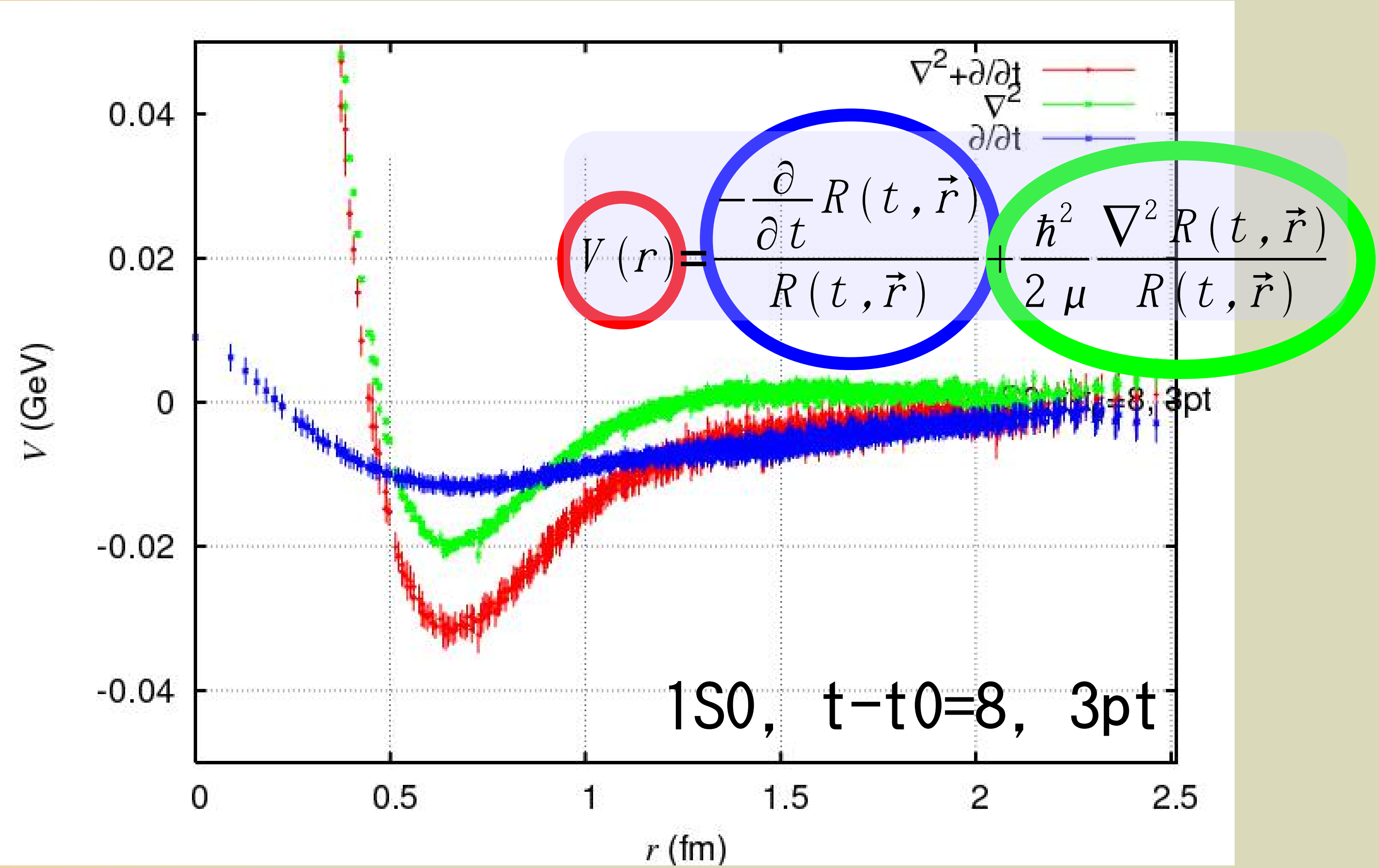


$V_T(\Lambda N;\ 3S1\text{-}3D1)$

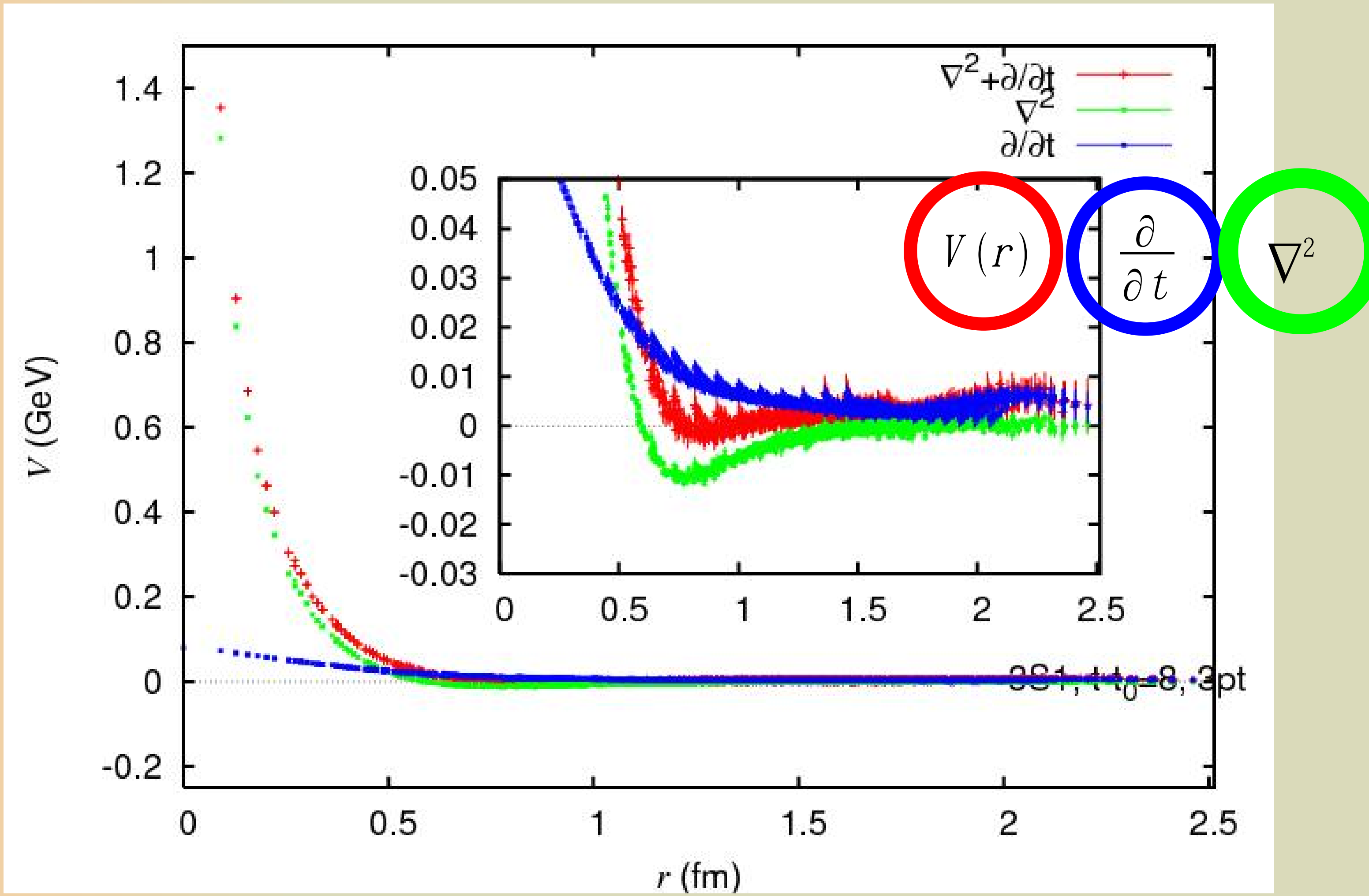


$\Sigma N(I=3/2)$ potential

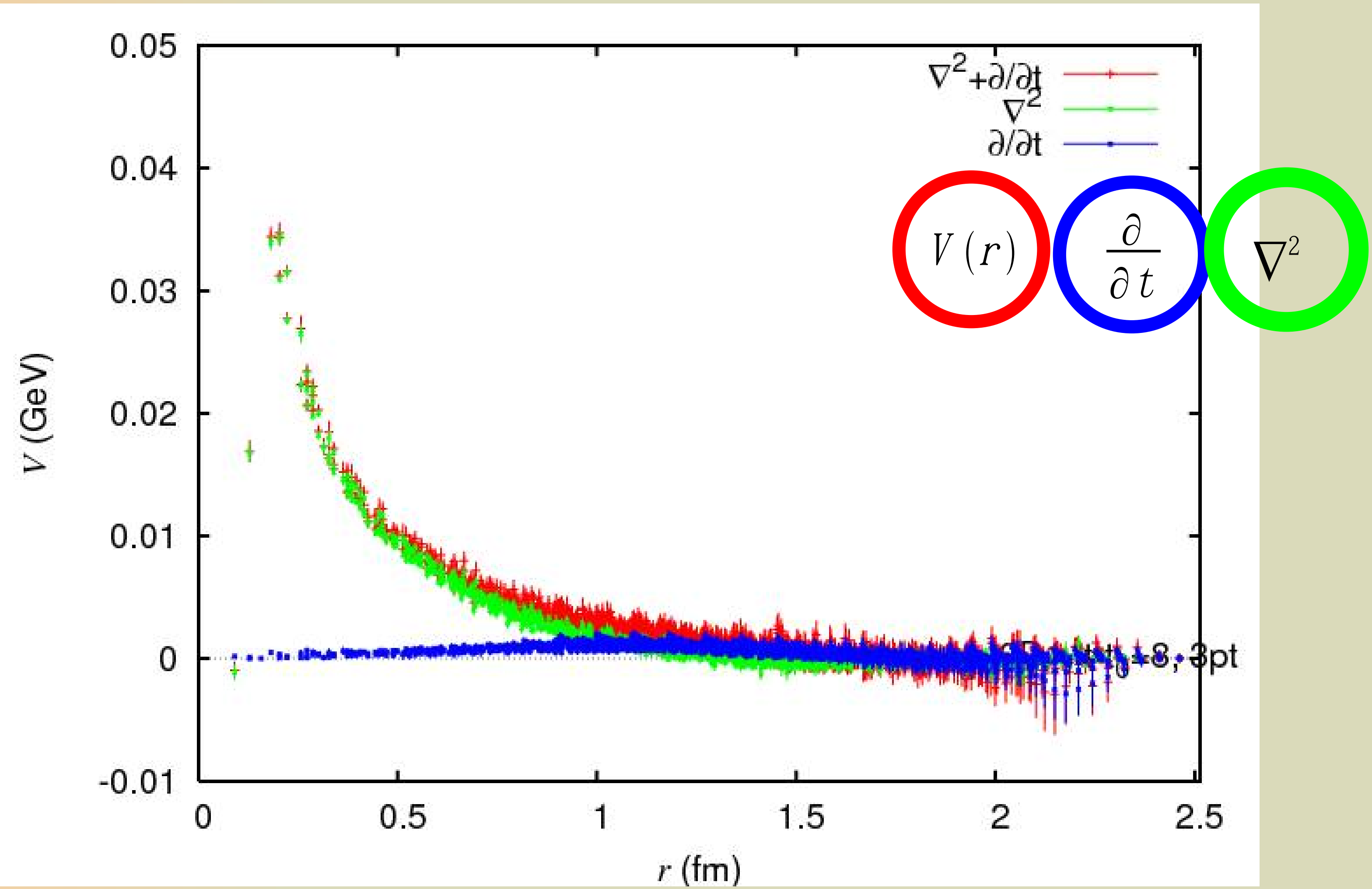
$V_C(\Sigma N(I=3/2);\ 1S0)$



V_c(ΣN(I=3/2); 3S1-3D1)



$V_T(\Sigma N(I=3/2);\ 3S1\text{-}3D1)$



H-dibaryon at the flavor SU(3) points

Lattice QCD Study of the H-dibaryon in Flavor SU(3) Limit

T. Inoue[A], N. Ishii[B], H. Nemura[C], K. Sasaki[C], T. Doi[D], K. Murano[E], Y. Ikeda[F], S. Aoki[C,B], T. Hatsuda[G,E]

Nihon Univ.[A], CCS U.Tsukuba[B], Univ. Tsukuba[C], CNS U. Tokyo[D], Riken Nishina[E], Tokyo Inst. Tech.[F] Univ. Tokyo[G]

HAL QCD Collaboration

- Introduction
- Problem
- New method
- Setup, Results
- Summary, Outlook

Inoue, (HAL QCD, JPS meeting, 2011).

> H-dibaryon at SU(3) points

Lattice, Action and Facility

β	a [fm]	Lattice	L [fm]
1.83	0.121(2)	32^3 x 32	3.87

- Renormalization group improved Iwasaki gauge and Non-perturbatively O(a) improved Wilson quark
- We thank K.-I. Ishikawa and the PACS-CS group for providing their DDHMC/PHMC code to generate gauge configuration, and the Columbia Physics System for their lattice QCD simulation code.
- We enhance S/N of data by averaging on 4x4=16 source, and forward/backward propagation in time.
- All numerical computation carried at the supercomputer system T2K-Tsukuba.

Inoue, (HAL QCD, 2011).

> H-dibaryon at SU(3) points

Five ensembles

TI et.al. Phys. Rev. Lett. 106, 162002(2011)

K_uds	N_cfg	M_P.S. [MeV]	M_Vec [MeV]	M_Bar [MeV]	
0.13660	420	1170.9(7)	1510.4(0.9)	2274(2)	New
0.13710	360	1015.2(6)	1360.6(1.1)	2031(2)	add cfg
0.13760	480	836.8(5)	1188.9(0.9)	1749(1)	add cfg
0.13800	360	672.3(6)	1027.6(1.0)	1484(2)	add cfg
0.13840	600	468.9(8)	830.6(1.5)	1163(2)	New

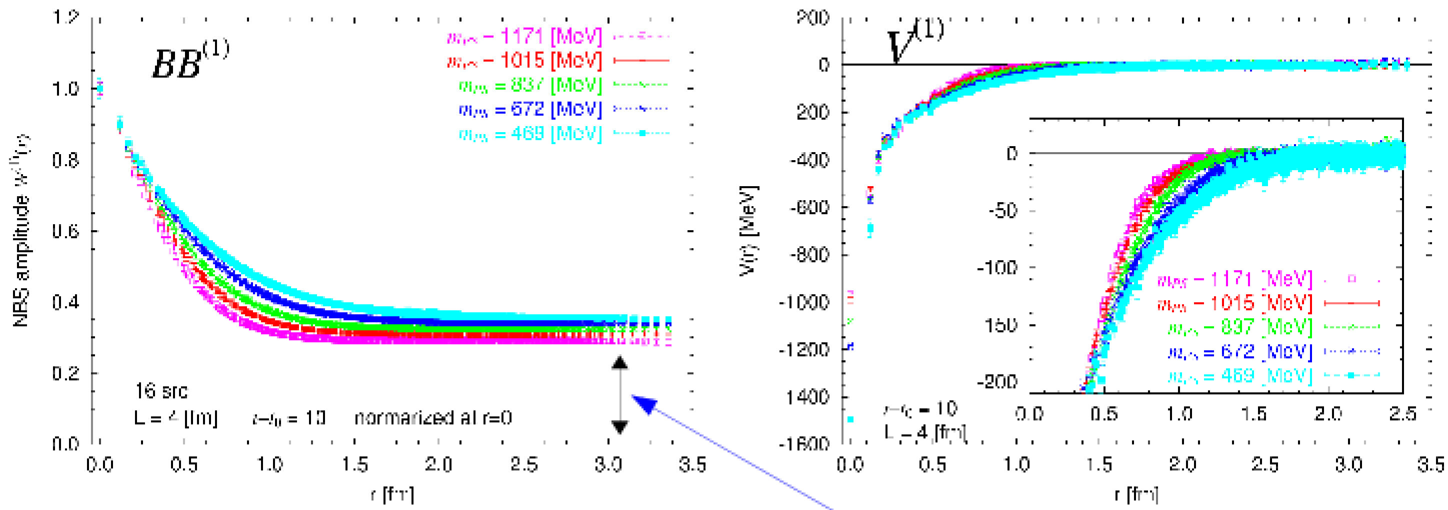
relative to

- We've made five ensembles with different hopping parameter, (=quark mass) corresponding to M_{PS} = 1.17 [GeV] to 470 [MeV].
- With lightest quark (bottom row of the table),
  - p.s. meson is a little lighter than the physical kaon.
  - baryon is a little lighter than the physical sigma baryon.
- Now, the simulated hadron world is not so far from the real world, although the $SU(3)_F$ breaking is not taken into account.

Inoue, (HAL QCD, 2011).

> H-dibaryon at SU(3) points

NBS w.f. & potential

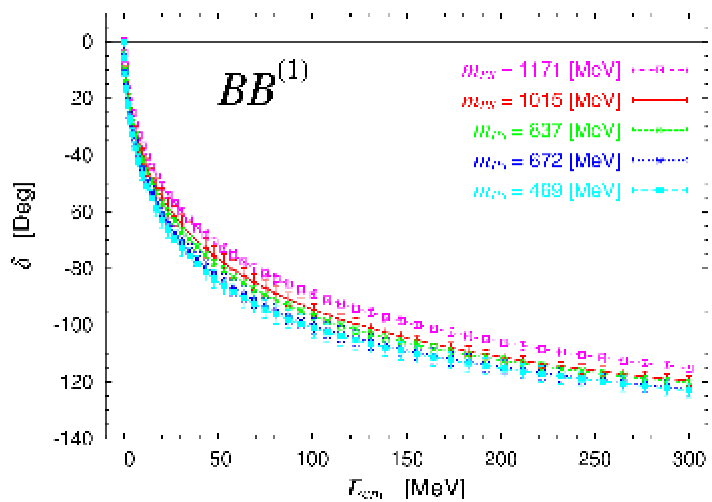


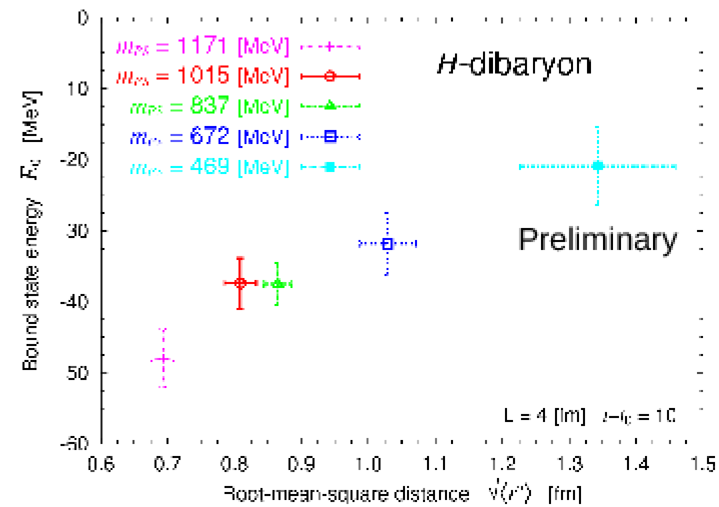
- Left: Measured NBS w.f. of the 1F channel
  - The finite value at large distance is excited states contribution as well as a finite volume effect. (demonstrated in later)
- Right: Extracted potential of the 1F channel
  - $V^{(1)}(r)$ become more attractive as quark mass decrease.

Inoue, (HAL QCD, 2011).

> H-dibaryon at SU(3) points

Observables





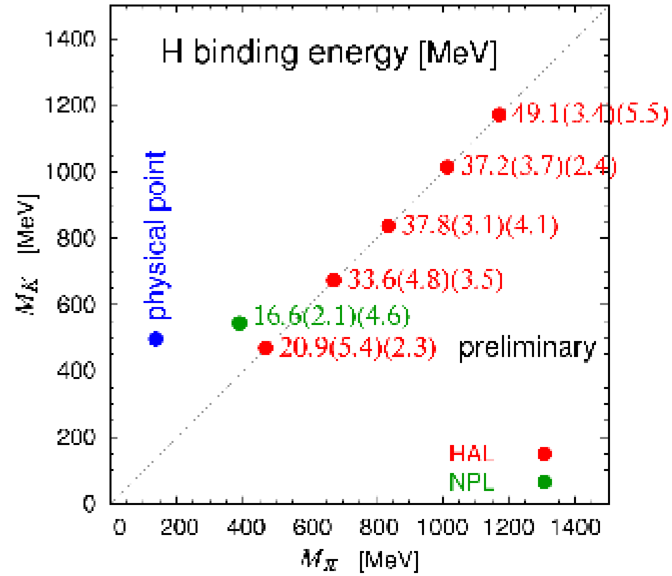
- Left: scattering phase shift v.s. E_{cm}
  - shows existence of one discrete state below threshold.
- Right: obtained ground state
  - which is 20 - 50 MeV below from free BB ie. 3q-3q.
  - This means that there is a 6-quark bound state in the 1_F channel.
  - A stable(bound) H-dibaryon exists in these $SU(3)_F$ limit world! 15

Inoue, (HAL QCD, 2011).

> H-dibaryon at SU(3) points

Summary & Outlook

- Plot of H binding energy from recent full QCD simulations.
  - SR. Beane etal [NPLQCD colla.] Phys. Rev. Lett. 106, 162001 (2011)
  - Obtained binding energy from the two groups looks consistent.
  - But, all data points are still away from the physical point.



- We'll continue this study and put more data on this plot.
- We'll take flavor SU(3) breaking soon. K.Sasaki on Thursday
- We'll get information of H-dibaryon in the physical world in near future.

Inoue, (HAL QCD, 2011).

ΛΛ-NΞ-ΣΣ(I=0), NΞ-ΛΣ-ΣΣ(I=1), and ΣΣ(I=2) potentials

Lattice QCD study of baryon-baryon interaction with strangeness S = -2

Kenji Sasaki (*University of Tsukuba*)

for HAL QCD collaboration

Hadrons to Atomic nuclei
HAL
from Lattice QCD

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

S. Aoki (*Univ. of Tsukuba*)	**T. Doi** (*Univ. of Tokyo*)	**T. Hatsuda** (*RIKEN*)	**Y. Ikeda** (*Tokyo Inst. Tech.*)
T. Inoue (*Nihon Univ.*)	**N. Ishii** (*Univ. of Tsukuba*)	**K. Murano** (*RIKEN*)	**H. Nemura** (*Tohoku Univ.*)

SASAKI, Kenji, (HAL QCD, JPS meeting, 2011).

> ΛΛ-NΞ-ΛΣ-ΣΣ potentials w/ SU(3) breaking

Numerical setup

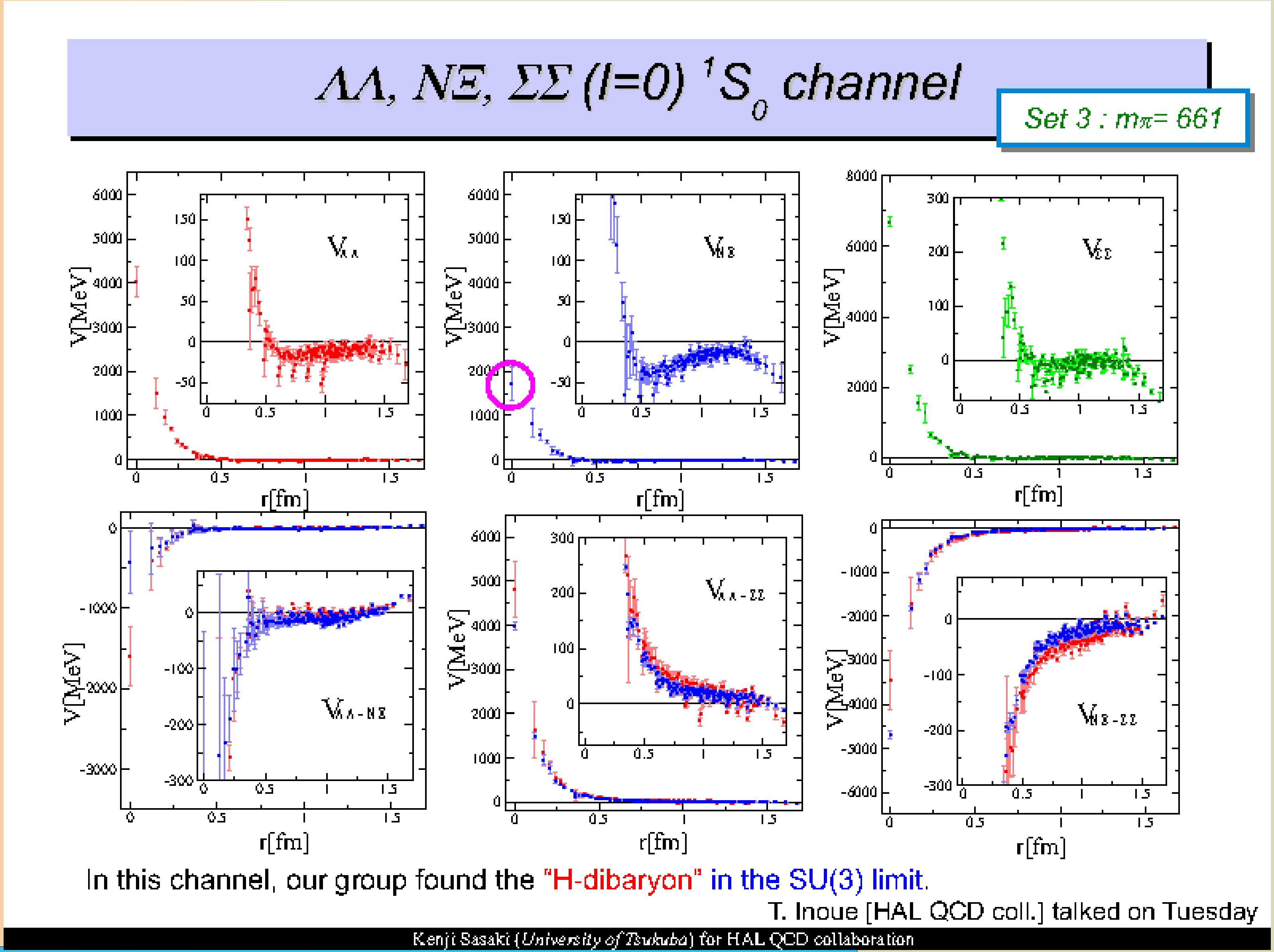
- 2+1 flavor gauge configurations by CP-PACS/JLQCD collaboration.
  - RG improved gauge action & *O(a)* improved clover quark action
  - β = 1.83, a^{-1} = 1.632 [*GeV*], a = 0.1209 [*fm*]
  - 16^3x32 lattice, L = 1.934 [*fm*].
  - κ_{ud} = 0.13825, κ_s = 0.13710 was chosen (named Set3).
  - 350 / 800 configurations are used.
- Flat wall source is considered to produce S-wave B-B state.
  - Four shifted sources every 8 time-slices are considered to enhance the S/N ratio.
- The USQCD computer resources are used.
  - We acknowledge the USQCD for providing of computer resources.

	π	K	m_π/m_K	N	Λ	Σ	Ξ
Set3	661±1	768±1	0.860	1482±3	1557±3	1576±3	1640±3

In unit of MeV

SASAKI, Kenji, (HAL QCD, 2011).

> ΛΛ-NΞ-ΛΣ-ΣΣ potentials w/ SU(3) breaking



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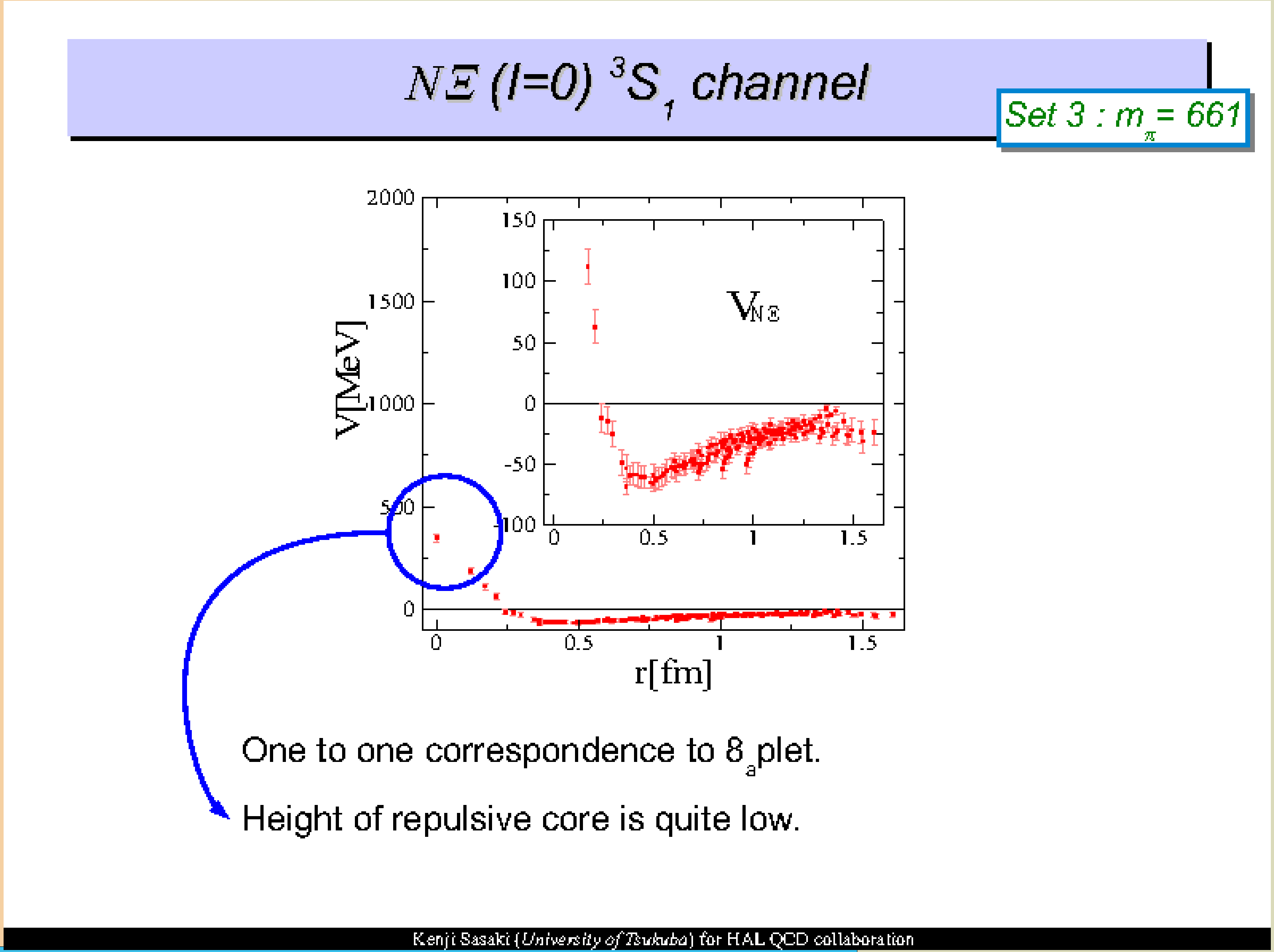

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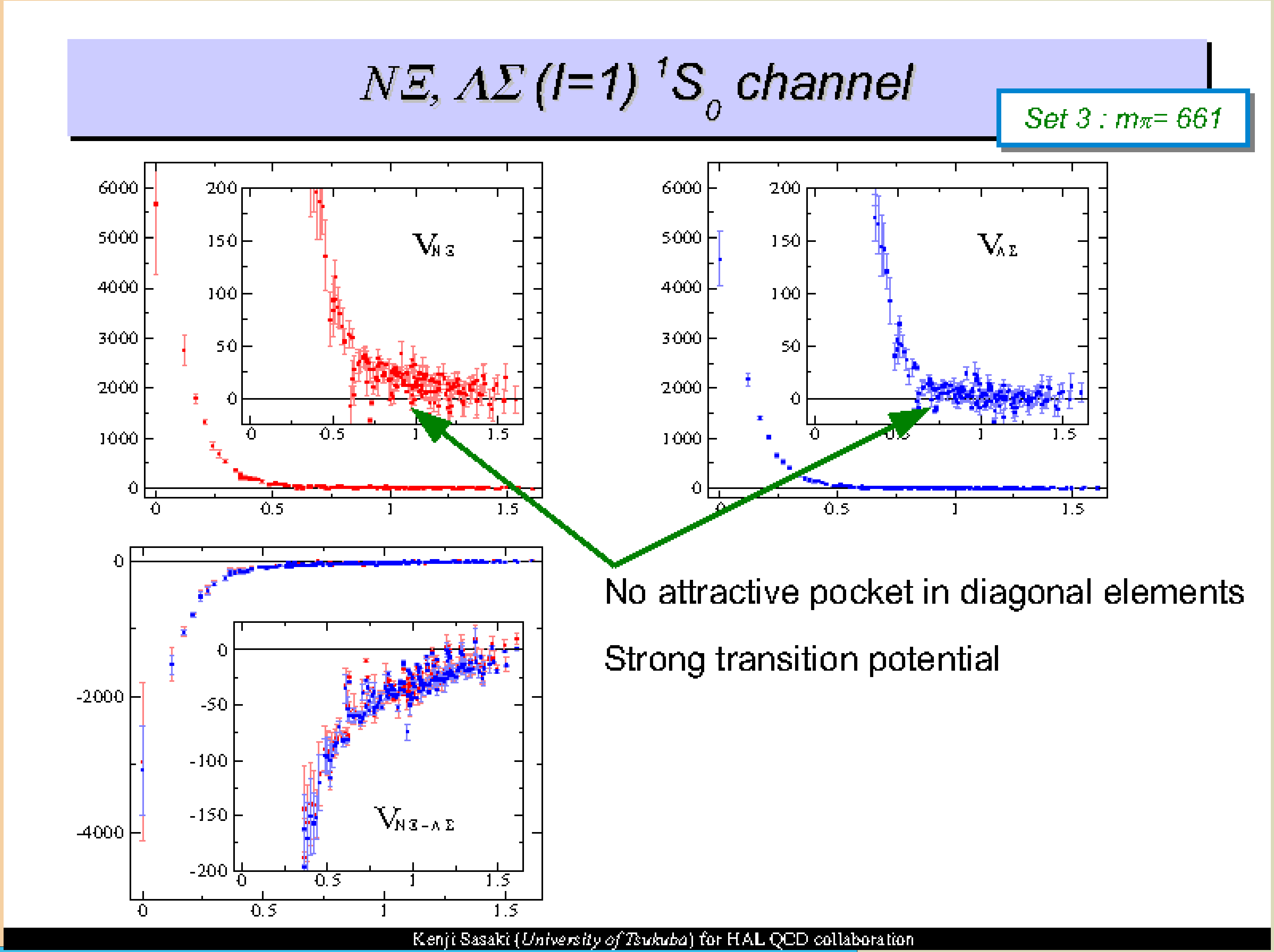

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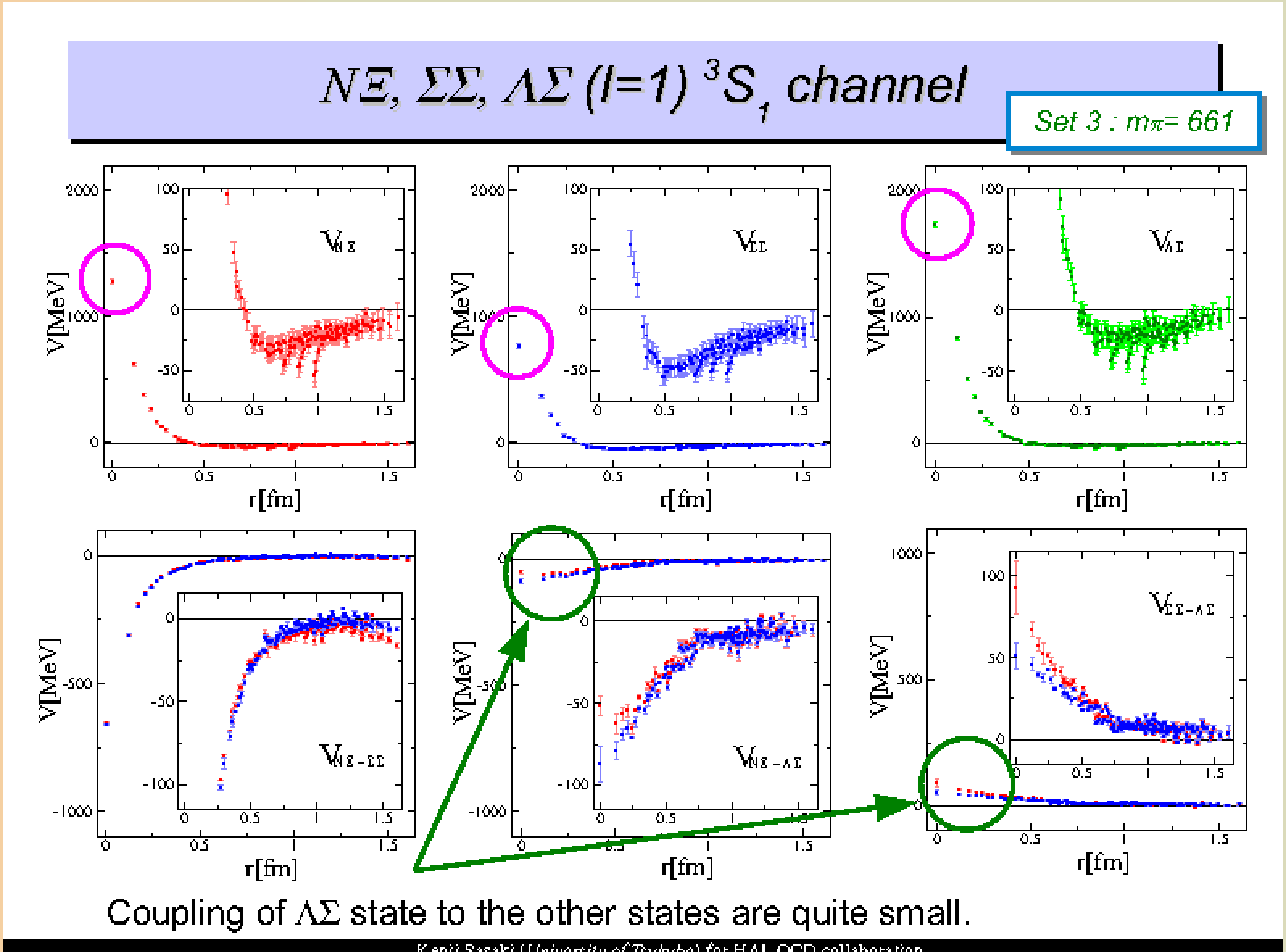
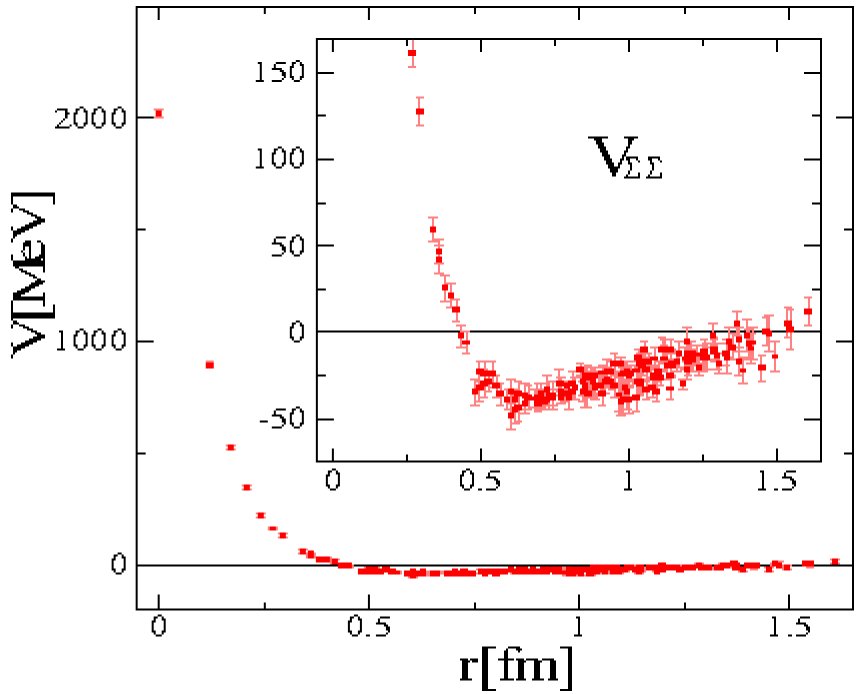


SASAKI, Kenji, (HAL QCD, 2011).

> ΛΛ-NΞ-ΛΣ-ΣΣ potentials w/ SU(3) breaking

ΣΣ (I=2) $^1S_0$ channel

Set 3 : m_π= 661



One to one correspondence to the 27plet in SU(3) irreducible representation

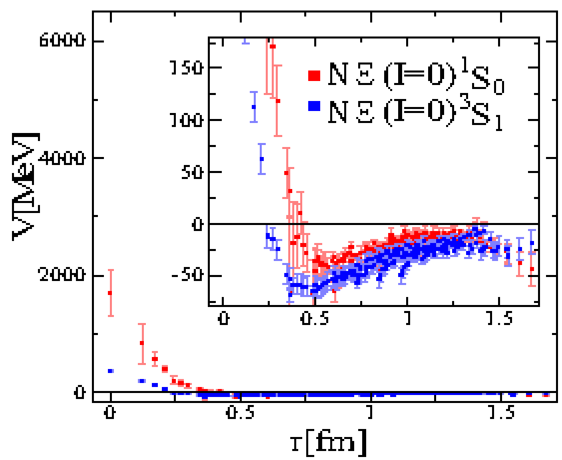
Similar behavior to the NN potential

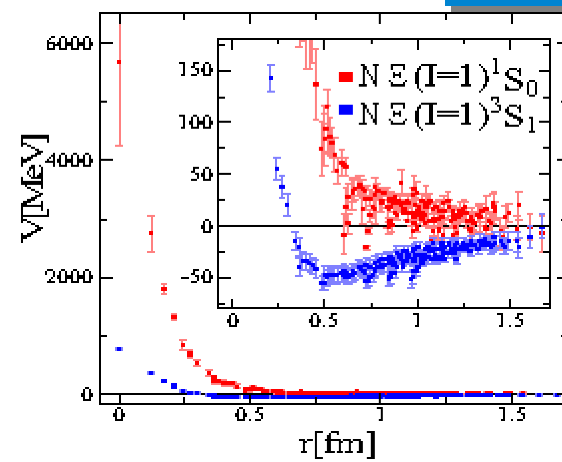
SASAKI, Kenji, (HAL QCD, 2011).

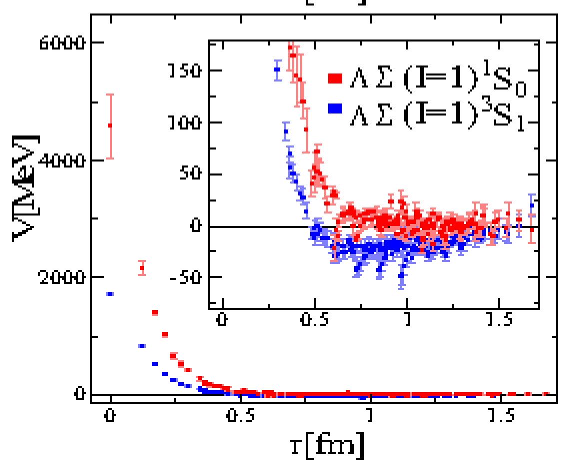
> ΛΛ-NΞ-ΛΣ-ΣΣ potentials w/ SU(3) breaking

Spin dependence of $N\Xi$, $\Lambda\Sigma$ potentials

Set 3 : m_π= 661







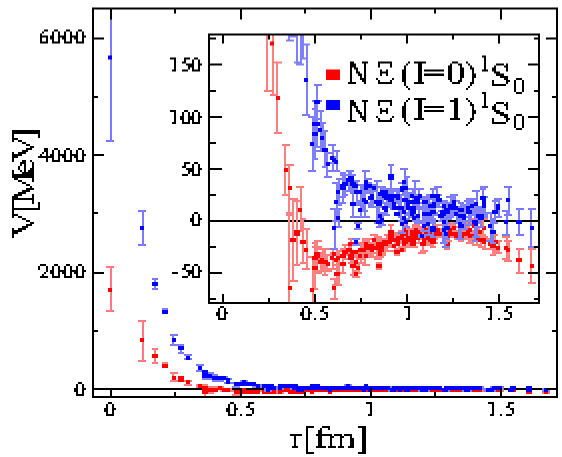
Spin triplet potentials are more attractive than the spin singlet potentials

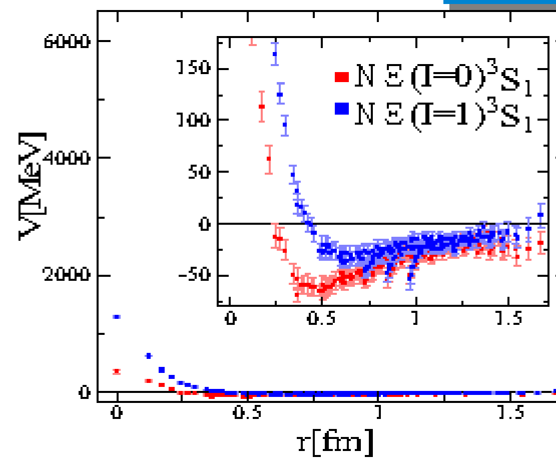
SASAKI, Kenji, (HAL QCD, 2011).

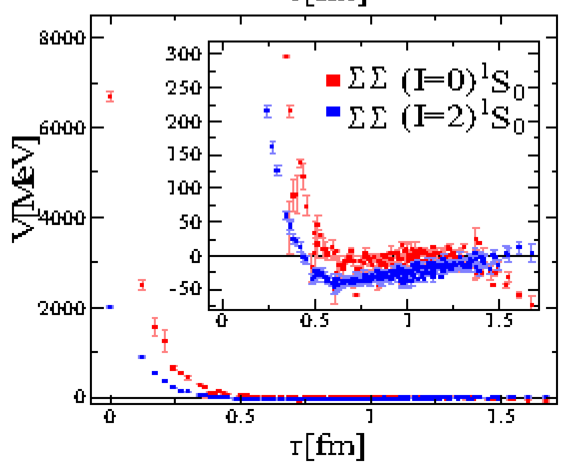
> ΛΛ-NΞ-ΛΣ-ΣΣ potentials w/ SU(3) breaking

Isospin dependence of $N\Xi$, $\Sigma\Sigma$ potentials

Set 3 : m_π= 661







In NΞ potentials the I=0 potentials are more attractive than the I=1 potentials.

The short range behavior of potentials are strongly depend on the choice of state.

Kenji Sasaki (*University of Tsukuba*) for HAL QCD collaboration

SASAKI, Kenji, (HAL QCD, 2011).

Summary:

- The lattice QCD study for Lambda-Nucleon and Sigma-nucleon(I=3/2) interactions.
- pΛ:
  - Central, tensor. For full QCD
  - Time-derivative terms enhance the attractive force.
  - Qualitatively similar to well-known nuclear forces.
    - Repulsive at short distance.
    - Attractive well at medium to long distance.
- NΣ(I=3/2):
  - Central, tensor. For full QCD
  - The 1S0 potential is similar to Lambda-N potential
  - The 3S1 potential is repulsive

Summary:

From Inoue's work:

> Bound H-dibaryons are found at the flavor SU(3) points.

> 32^3x32 lattice, L=3.87 [fm],

$m_{\text{ps-meson}}$=1.17-0.47GeV,

m_{Baryon}=2.27-1.16GeV $\rightarrow$ $B_{\text{H-dibaryon}}$=50-20MeV

From Sasaki's work:

> ΛΛ-NΞ-ΣΣ (I=0)

Weak ΛΛ-NΞ coupling

NΞ(3S1) seems to be (relatively) attractive among others in the S=-2 channel except for H-dibaryon channel.

> NΞ-ΛΣ-ΣΣ (I=1)

NΞ(I=1) is weakly attractive (3S1) or repulsive (1S0).

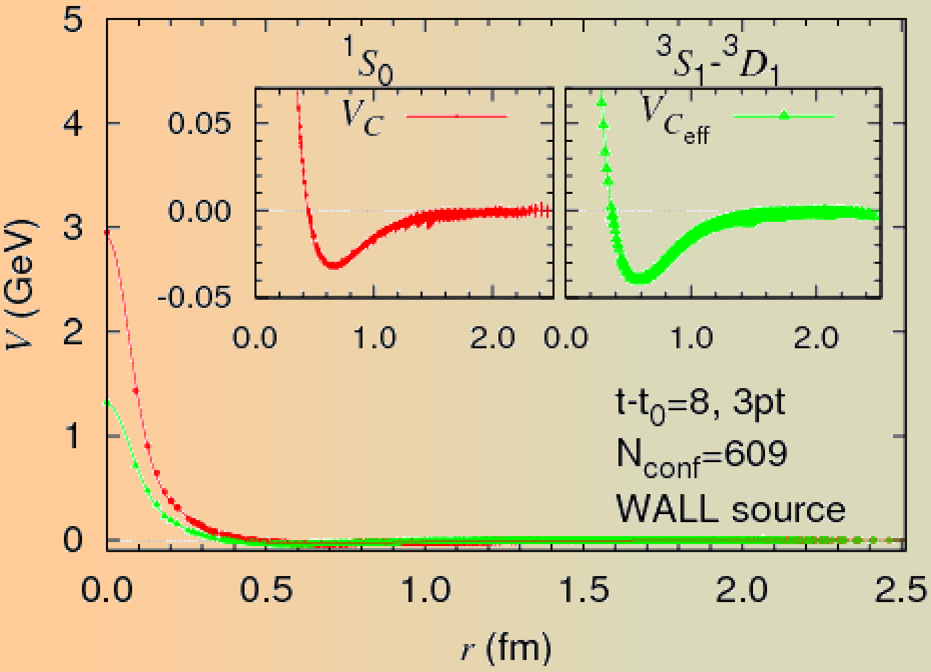
> ΣΣ (I=2)

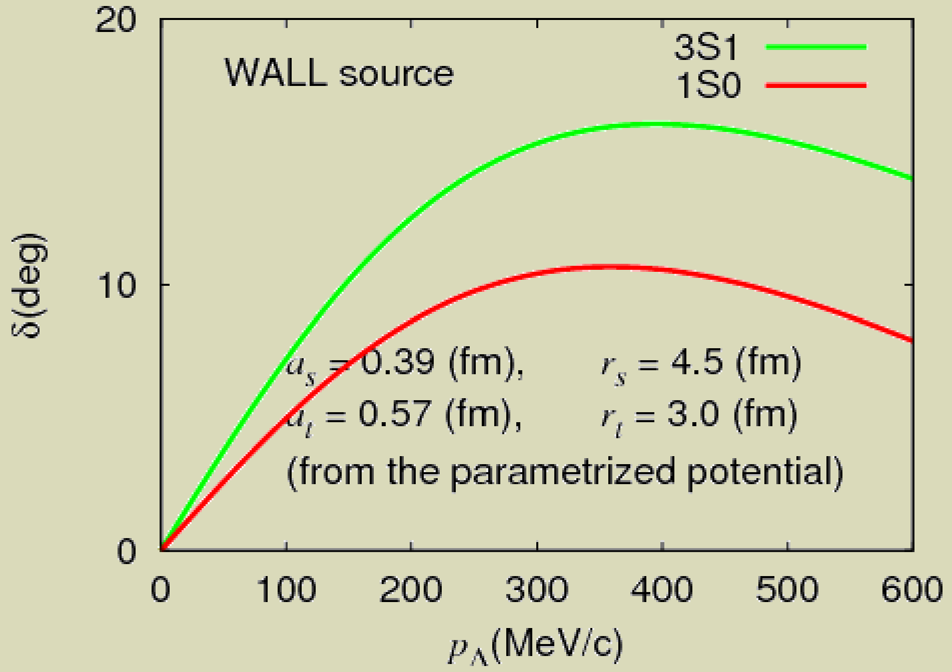
Similar to NN (1S0) due to the symmetry.

Outlook:

- Quark mass dependence.
- Scattering lengths.
    - spin-dependence.
    - Comparison with the hypernuclear data.
- Coupled-channel potential.

Proton-Lambda interaction (preliminary)





Outlook:

- ΣN(I=3/2, 3S1) repulsive.

- H-dibaryon:
  - Lattice QCD shows that a bound H-dibaryon at flavor SU(3) limit.
  - We need to further study towards the physical point.

- Ξ-hypernucei?