MMC(metallic magnetic calorimeter) as a low temperature detector and its energy resolution

Weekly meeting 2018/09/10

Outline

1. Detection mechanism of MMC detector

2. Optimal parameters

3. Energy resolution

4. Summary

1. Detection mechanism with MMC detector

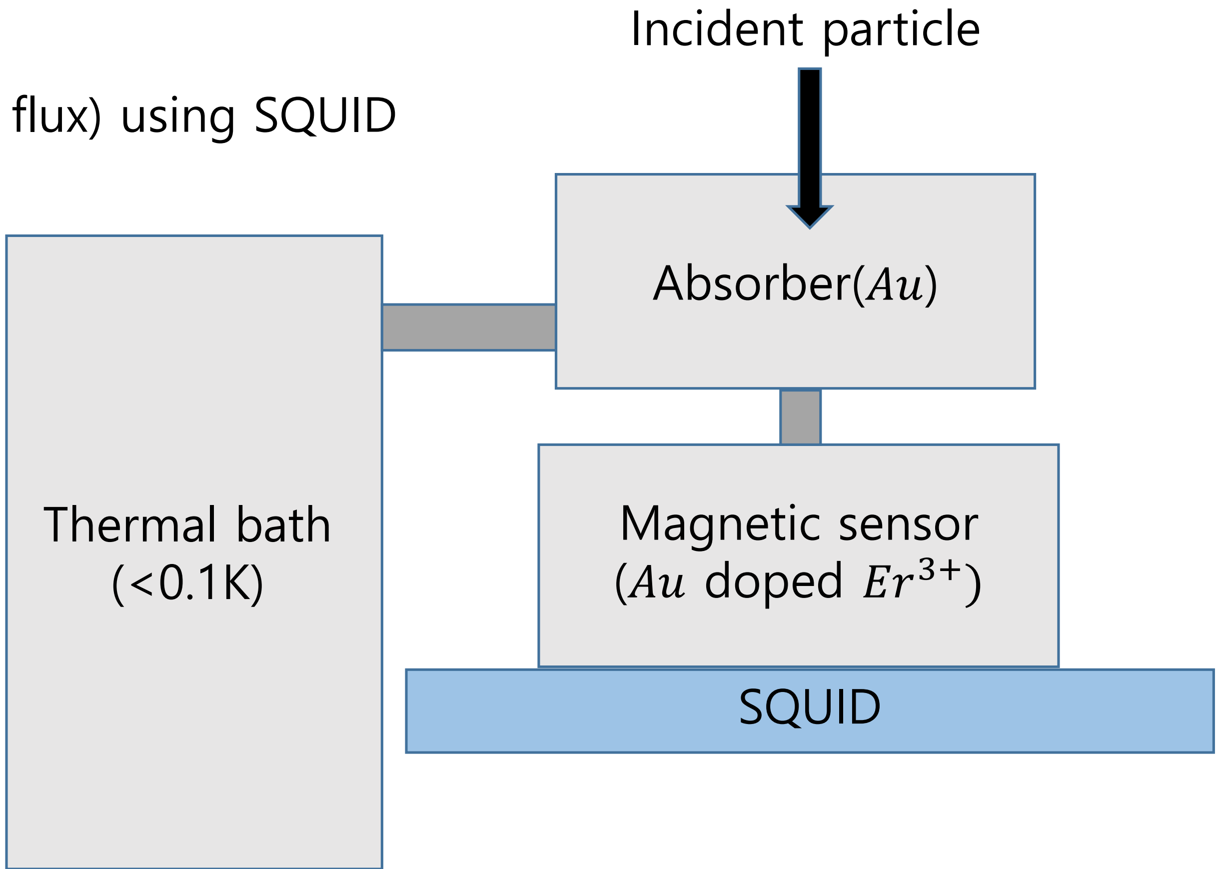
- MMC(metallic magnetic calorimeter)

- Deposited energy fluctuates the total magnetic moment(sensor) embedded in the metallic medium

- Measure this fluctuating magnetic moment(magnetic flux) using SQUID

- Absorber : For the fast response(w sensor), use gold (rich conduction electron)(but large heat capacity)

- Magnetic sensor : Er^{3+} ions in place of gold lattice (small interaction between moments)



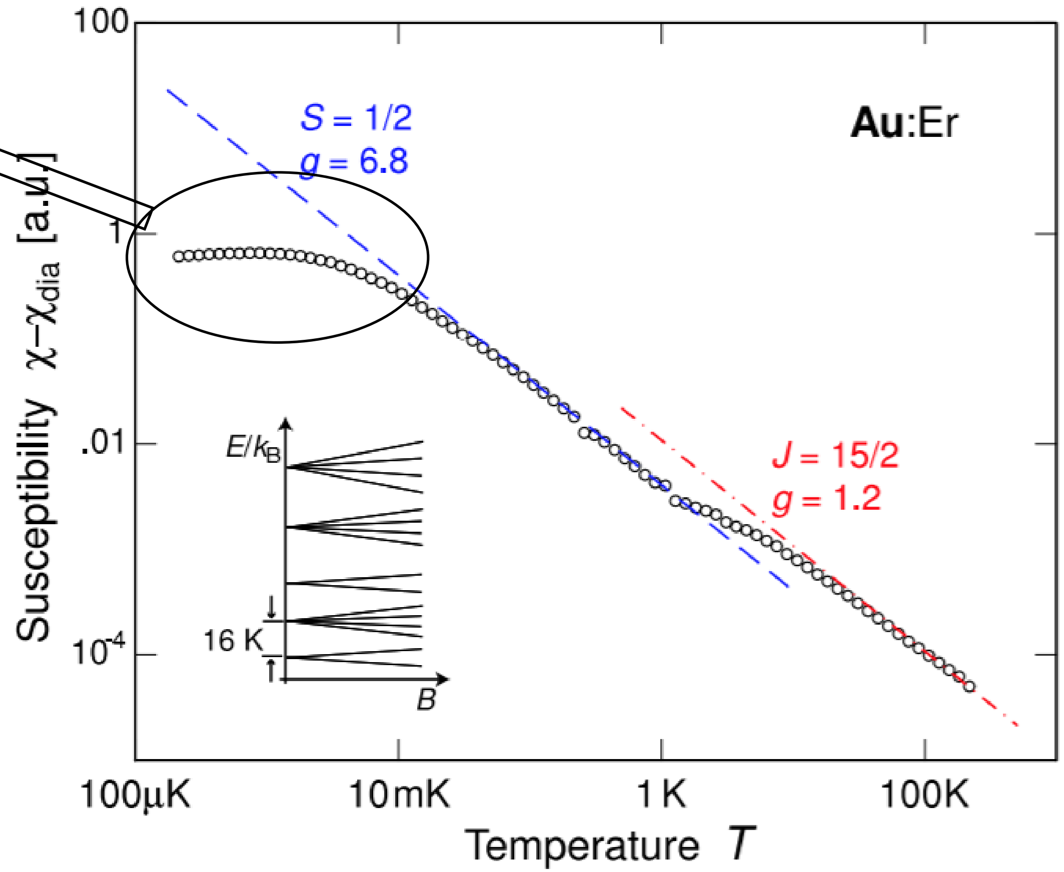
1.1 magnetic sensor (Au: Er)

- Thermal equilibrium state can be described by spin ½ two-level system with g = 6.8

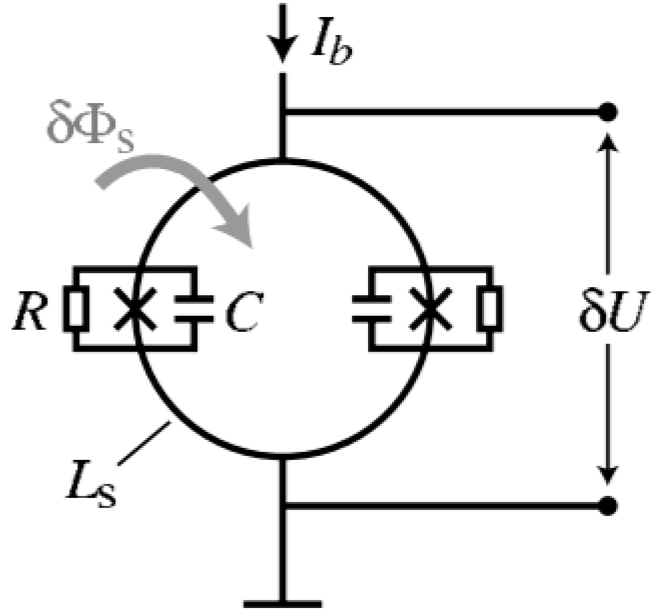
- Heat capacity of magnetic moments : $C_s = Nk_B\left(\frac{g\mu_B B}{k_B T}\right)^2 \frac{e^{\frac{g\mu_B B}{k_B T}}}{\left(e^{\frac{g\mu_B B}{k_B T}}+1\right)^2}$

- Interaction between magnetic moments has to be accounted below temperature T ~ 50mK

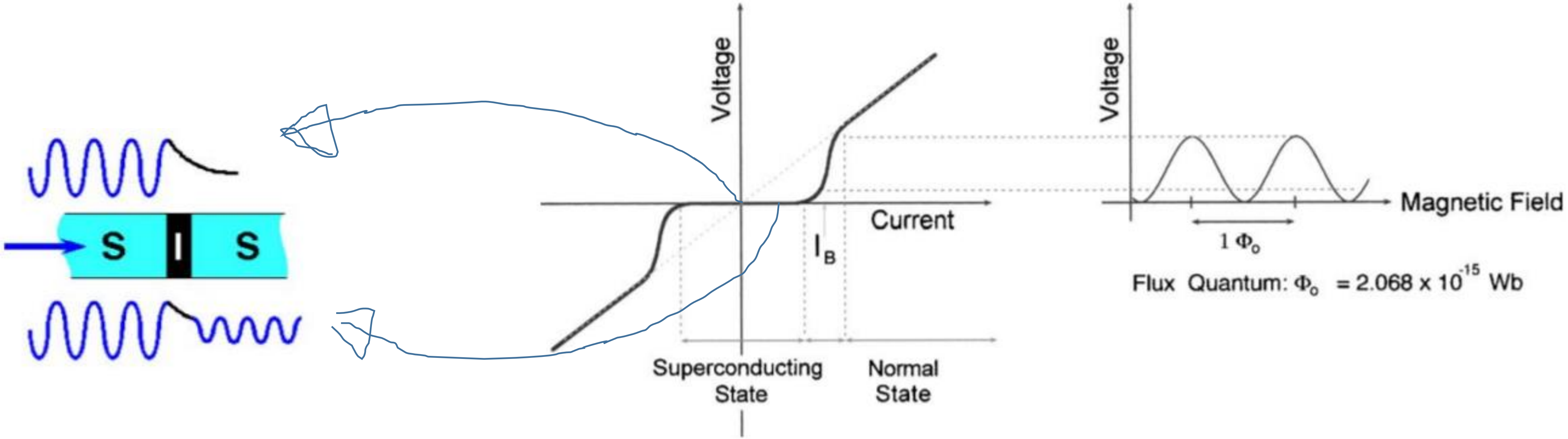
- It reduce the response of magnetic flux and increases the Heat capacity



1.2 dc-SQUID



- It reads the magnetic flux through the SQUID loop
- Set the bias current I_B slightly above the twice the saturated current($2I_0$) so that any additional current make fermi level difference(voltage) between two connected superconductors
- As the flux increases, inducted current oscillates because magnetic flux inside the loop has to be integer multiple of $\Phi_0 = h/2e$
- Resulting voltage drop is sensed by additional circuit



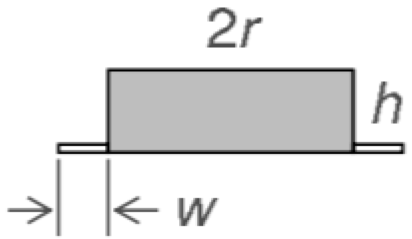
2. Optimal parameters

- Consider flux noise due to the squid(electron thermal motion, …) S_{ϕ_s}, it results in the energy sensitivity $\epsilon_s = \frac{S_{\phi_s}}{2L_s}$ (L_s: $inductance\ of\ squid\ loop$) $\langle(\Delta\phi)^2\rangle = \int_0^\infty S_\phi df$
- Assume that flux noise is frequency independent and set $\epsilon_s \sim 25\hbar$
- Maximize (Flux responsivity to incident energy) / (flux noise) = $\frac{\delta\phi}{\delta E}/\sqrt{S_{\phi_s}} \propto \frac{\delta\phi}{\delta E}/\sqrt{L_s}$
- Assume the geometry of page3 (cylindrical magnetic sensor, surrounded by SQUID loop at one end)
- Use $\delta m = V\left(\frac{\partial M}{\partial T}\right)\left(\frac{\partial E}{C_a+cV}\right)$

$(C_a : heat\ capacity\ of\ absorber\ and\ electrons\ in\ sensor, V : volume\ of\ sensor, c : heat\ capacity\ of\ magnetic\ moments)$

$$\frac{\delta\phi}{\delta E}/\sqrt{L_s} = (\text{geometry factor}(h/r))*\frac{\sqrt{V}}{cV+C_a}*\frac{\partial M}{\partial T}\bigg|_{V\ opt} = (\text{geometry factor}(h/r))*\frac{1}{2\sqrt{C_a}}*\frac{1}{\sqrt{c}}\frac{\partial M}{\partial T}\bigg|_{x,B\ opt} = \frac{0.042}{\sqrt{C_a T}}$$

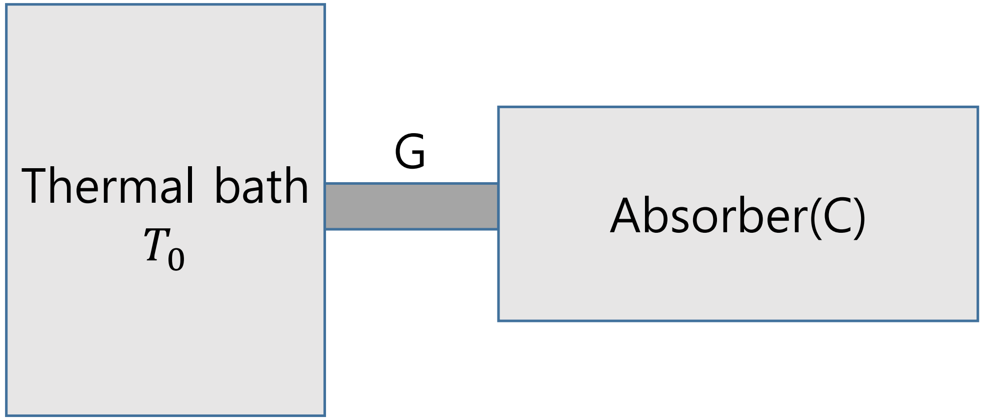
Parameters, that maximize $\mathcal{S} = (\delta\Phi/\delta E)/\sqrt{L}$ for cylindrical sensors				Example: Au:Er, $T = 0.05$ K, $C_a = 1\times10^{-12}$ J/K
B_{opt}	=	$2.1\,\mathrm{T\,K^{-1}}$	$\times\ Tg^{-1}$	15 mT
x_{opt}	=	$10.3\,\mathrm{K^{-1}}$	$\times\ Tg^{-2}\alpha^{-1}$	2200 ppm
r_{opt}	=	$0.64\,\mathrm{cm}\frac{\mathrm{K}^{2/3}}{\mathrm{J}^{1/3}}$	$\times\ (C_a g^2 \alpha T^{-1})^{1/3}$	10.7 µm
h_{opt}	=	0.53	$\times\ r_{opt}$	5.7 µm
$\mathcal{S}_{opt}$	=	0.093	$\times\ (C_a \alpha T)^{-1/2}$	



3. Energy resolution

Now consider time structure of signal and noises (consider frequency dependence)

3.1 thermodynamic energy fluctuation



- thermodynamic noise of frequency $f = \frac{w}{2\pi}$

- $C\frac{dT}{dt} = P_{in} - G(T - T_0)$
- $\langle(\Delta E)^2\rangle = \langle(C\Delta T)^2\rangle = k_B C {T_0}^2 = \int_0^\infty S_E(f)df$ $\qquad S_{\phi_s}$
- $P_{in}=P_0+P_1e^{iwt}$ & $T=T_0+T_1e^{iwt}$ ➡ $\frac{S_P(f)}{S_E(f)/C^2} = \frac{{P_1}^2}{{T_1}^2} = G^2 + C^2(2\pi f)^2$

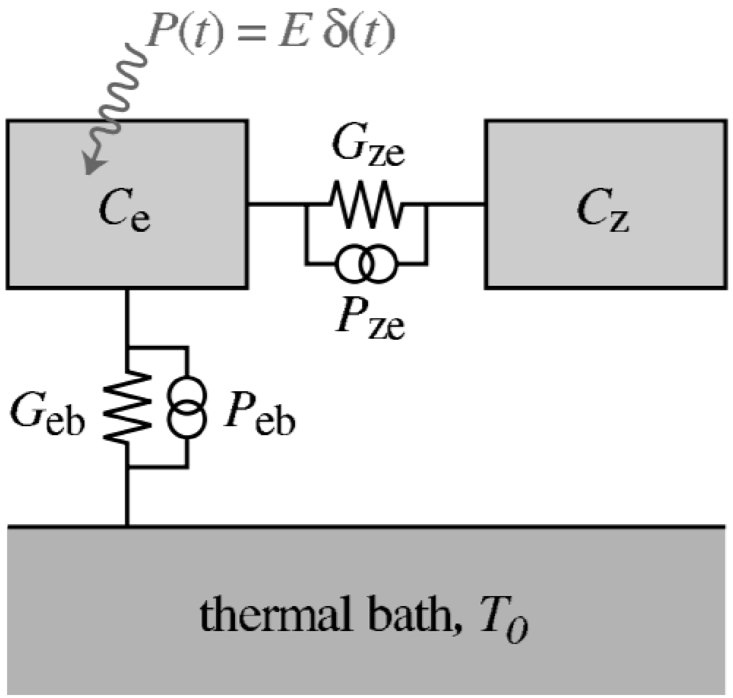
If we assume that fluctuation of heat flow from thermal bath is frequency independent, $S_P(f) = S_P$
then we can get

$$S_E(f) = k_B C {T_0}^2 \frac{4\tau}{1+(2\pi f\tau)^2}\ ,\ \tau = C/G$$

- Signal

- $P_{in}= E_0\delta(t)$ ➡ $E(t) = E_0 e^{-t/\tau}$ ->Fourier spectrum : $E(f) = E_0 \frac{\tau}{\sqrt{1+(2\pi f\tau)^2}}$

- In MMC case, there are two source of energy fluctuation between magnetic moment and conduction electron, and thermal bath.. -> can expect there will be to characteristic time



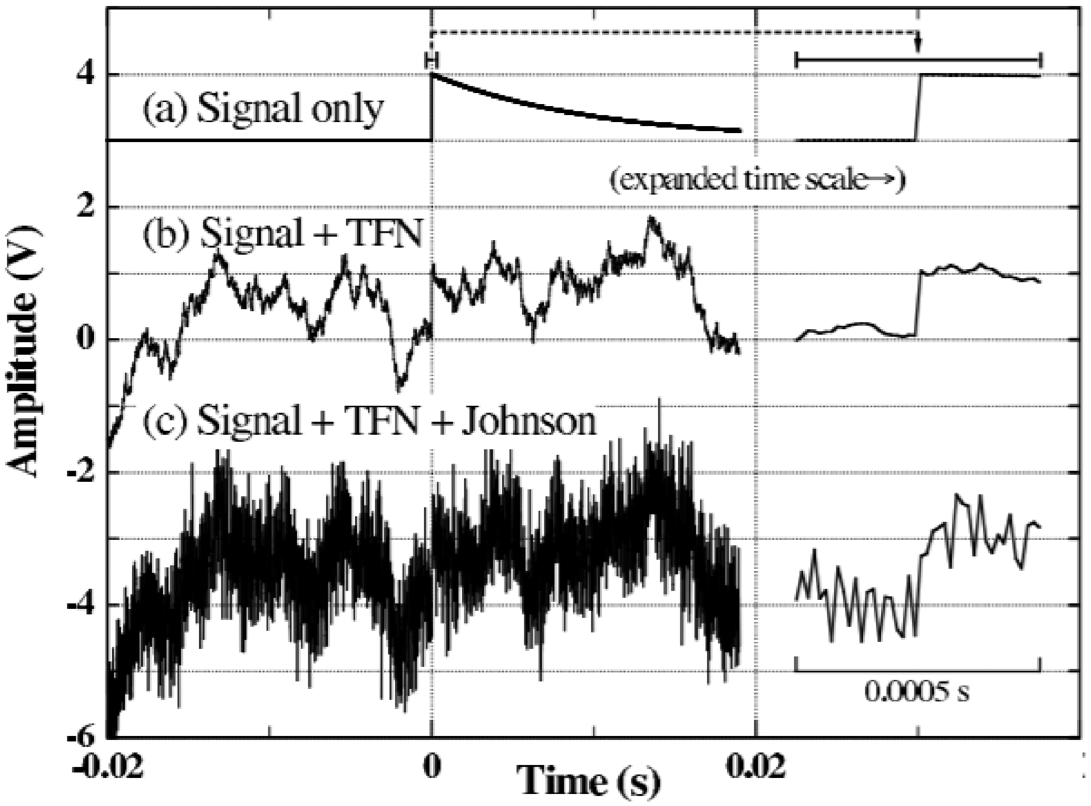
- $S_{E_Z}(f) = k_B C_Z {T_0}^2 \left((1-\beta)\frac{4\tau_0}{1+(2\pi f\tau_0)^2} + \beta\frac{4\tau_1}{1+(2\pi f\tau_1)^2} \right)$, $\beta = \frac{C_Z}{C_e + C_Z}$

$$\tau_1 \sim \frac{C_e + C_Z}{G_{eb}} \;\; \& \;\; \tau_0 \sim \frac{(C_e C_Z)/(C_e + C_Z)}{G_{Ze}}$$

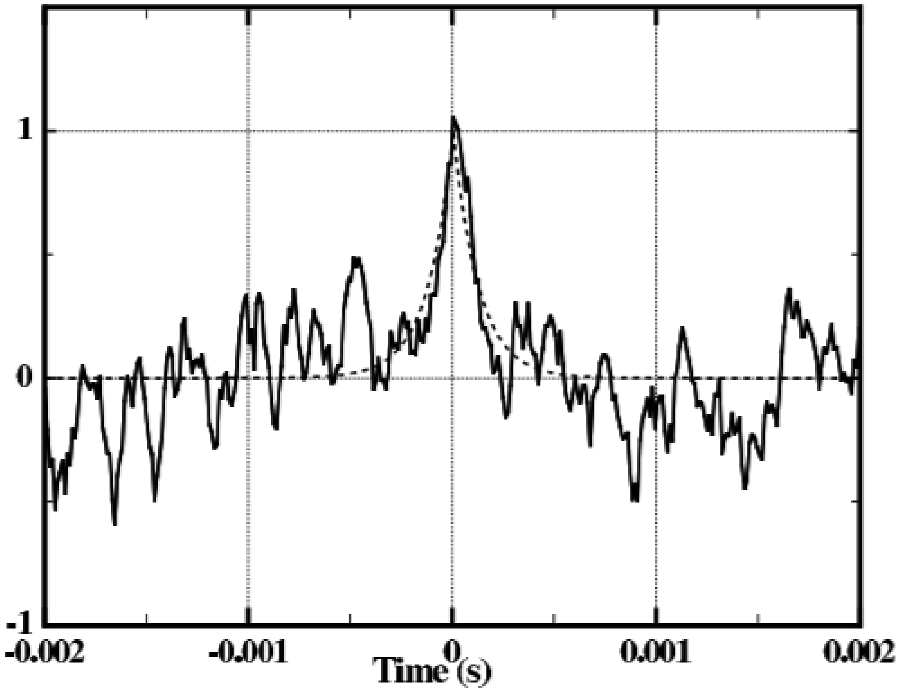
- $E_Z(t) = E_0\beta\left(e^{-t/\tau_1} - e^{-t/\tau_0}\right) \Rightarrow E_Z(f) = E_0 \frac{\tau_1}{\sqrt{1+(2\pi f\tau_0)^2}\sqrt{1+(2\pi f\tau_1)^2}}$

Noise equivalent power(NEP) : $\sqrt{S_{E_Z}(f)}\frac{E_0}{E_Z(f)}$

Ultimate energy resolution after optimal filter(maximizing $E/\Delta E$) : $\Delta E_{rms} = \left(\int_0^\infty \frac{4df}{NEP^2}\right)^{-1/2}$



Optimal filter



Result

$$\Delta E_{rms} = \sqrt{4k_B C_e {T_0}^2 \left[\frac{G_{eb}}{G_{Ze}} + \left(\frac{G_{eb}}{G_{Ze}}\right)^2\right]^{1/4}} = \sqrt{8k_B C_e {T_0}^2 \left[\frac{\tau_0}{\tau_1}\right]^{1/4}}$$

$\tau_1 \gg \tau_0$ $\qquad$ $C_Z = C_e$

- For $240 \times 240 \times 5\mu m^3$ gold absorber, $C_e \sim 1.0pJ/K$ and rise time $\tau_0 = 1\mu s$, decay time $\tau_1 = 1ms$ at $T_0 = 50mK$ -> $\boldsymbol{\Delta E_{FWHM} = 1.4eV}$

3.2 Energy resolution by squid noise

$$\text{NEP} = \sqrt{S_{\phi_s}}\frac{\delta E}{\delta\phi}\frac{E_0}{E_Z(f)} = \sqrt{2\epsilon_s L_s}\frac{\delta E}{\delta\phi}\frac{E_0}{E_Z(f)} = \sqrt{2\epsilon_s}\frac{\sqrt{C_e T_0}}{0.042}\frac{\tau_1}{\sqrt{1+(2\pi f\tau_0)^2}\sqrt{1+(2\pi f\tau_1)^2}}, \quad \epsilon_s = 25\hbar(expected\ value?)$$

$\boldsymbol{\Delta E_{FWHM} = 0.18eV}$

3. Summary

- Thermodynamic energy fluctuation is a dominant source of energy resolution degradation
- Making absorber as small as possible will be effective to improvement of energy resolution ($\Delta\boldsymbol{E}_{\boldsymbol{FWHM}} \propto \sqrt{V_{absorber}}$)
- More weaker thermal coupling to heat bath

⇒ advantageous to $\Delta\boldsymbol{E}_{\boldsymbol{FWHM}}$ but opposite to pile-up resolving

⇒ need a new technique to pile-up resolving : heat switch??